R-22

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

MRCET CAMPUS

(AUTONOMOUS INSTITUTION – UGC, GOVT. OF INDIA)

Department of AERONAUTICAL ENGINEERING

AIRCRAFT STRUCTURES

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AIRCRAFT STRUCTURES

B.TECH (R-22 Regulation)
(III YEAR – I SEM)
(2025-26)

DEPARTMENT AERONAUTCAL ENGINEERING

MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

(Autonomous Institution – UGC, Govt. of India)
Recognized under 2(f) and 12 (B) of UGC ACT 1956
(Affiliated to JNTUH, Hyderabad, Approved by AICTE - Accredited by NBA & NAAC – 'A' Grade - ISO 9001:2015 Certified)
Maisammaguda, Dhulapally (Post Via. Hakimpet), Secunderabad – 500100, Telangana State, India

Department of AERONAUTICAL ENGIERRING

Vision

- Department of Aeronautical Engineering aims to be indispensable source in Aeronautical Engineering which has a zeal to provide the value driven platform for the students to acquire knowledge and empower themselves to shoulder higher responsibility in building a strong nation.

Mission

- The primary mission of the department is to promote engineering education and research. To strive consistently to provide quality education, keeping in pace with time and technology. Department passions to integrate the intellectual, spiritual, ethical, and social development of the students for shaping them into dynamic engineers.

QUALITY POLICY

- Impart up-to date knowledge to the students in Aeronautical area to make them quality engineers. Make the students experience the applications on quality equipment and tools. Provide systems, resources, and training opportunities to achieve continuous improvement. Maintain global standards in education, training, and services.

PROGRAM OUTCOMES
(PO's)

Engineering Graduates will be able to:

- Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
- Problem analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
- Design / development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal and environmental considerations.
- Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
- Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
- The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
- Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
- Ethics: Apply ethical principles and commit to professional ethics and

responsibilities and norms of the engineering practice. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

- Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
- Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
- Life- long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAM EDUCATIONAL OBJECTIVES – Aeronautical Engineering

- PEO1 (PROFESSIONALISM & CITIZENSHIP): To create and sustain a community of learningin which students acquire knowledge and learn to apply it professionally with due consideration for ethical, ecological and economic issues.
- PEO2 (TECHNICAL ACCOMPLISHMENTS): To provide knowledge-based services to satisfy the needs of society and the industry by providing hands on experience in various technologies in core field.
- PEO3 (INVENTION, INNOVATION AND CREATIVITY): To make the students to design, experiment, analyze, and interpret in the core field with the help of other multi- disciplinary concepts wherever applicable.
- PEO4 (PROFESSIONAL DEVELOPMENT): To educate the students to disseminate research findings with good soft skills and become a successful entrepreneur.
- PEO5 (HUMAN RESOURCE DEVELOPMENT): To graduate the students in building national capabilities in technology, education and research

PROGRAM SPECIFIC OUTCOMES – Aeronautical Engineering

- To mold students to become a professional with all necessary skills, personality and sound knowledge in basic and advance technological areas.
- To promote understanding of concepts and develop ability in design manufacture and maintenance of aircraft, aerospace vehicles and associated equipment and develop application capability of the concepts sciences to engineering design and processes.
- Understanding the current scenario in the field of aeronautics and acquire ability to apply knowledge of engineering, science and mathematics to design and conduct experiments inthe field of Aeronautical Engineering.
- 4. To develop leadership skills in our students necessary to shape the social, intellectual, business and technical worlds.

Dr. Ajith Raj R
Aug/2025

①

Aerospace Vehicle Structures - II
Aircraft Vehicle Structures

Unit - 2

Thin plate theory, Structural Instability: Analysis of thin rectangular plates subject to bending, twisting, distributed transverse load, Combined bending and in plane loading, local instability. Wagner beam analysis.

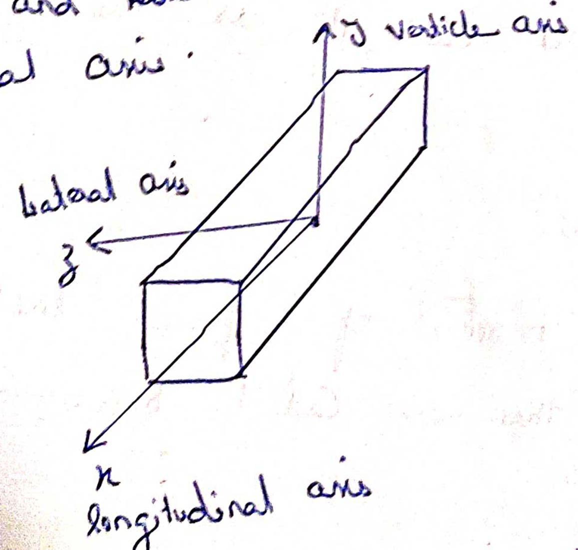
Thin plates:

Sheet of metal whose thickness is small but is capable of resisting bending.

Force:

Force when applied to an object tends to change its motion or its shape.

In structural engineering we have well defined cross sections and ~~these~~ members have a longitudinal and a lateral axis.



The force applied in the longitudinal axis of the member would tend to elongate (Tensile Force) or compress (compressive force) of the member.

the lateral axis would

A force applied in to slice of the member (shear force) or would try to bend the member (Bending Moment)

The amount of elongation, compression, or shearing is directly dependent on the magnitude of the force applied. The more is the force, more is the effect.

But the same is not the case with rotation. The same amount of force if applied at a greater distance would produce greater rotati

Moment of force:

Moment of force is the product of force and the distance.

Twisting moment:

If the moment force try to twist the member then we call it as twisting moment or torsion

Bending moment:

If the moment of force tries to bend the member, then we call it bending moment.

Assumptions:

1. Displacement of plate in the direction Parallel to z-axis is small when compared with the thickness.

2. The sections are plane before bending remain plane after bending.

3. The middle plane of the plate does not deform during bending and is therefore called a neutral plane.

4. Take neutral plane as the reference plane.

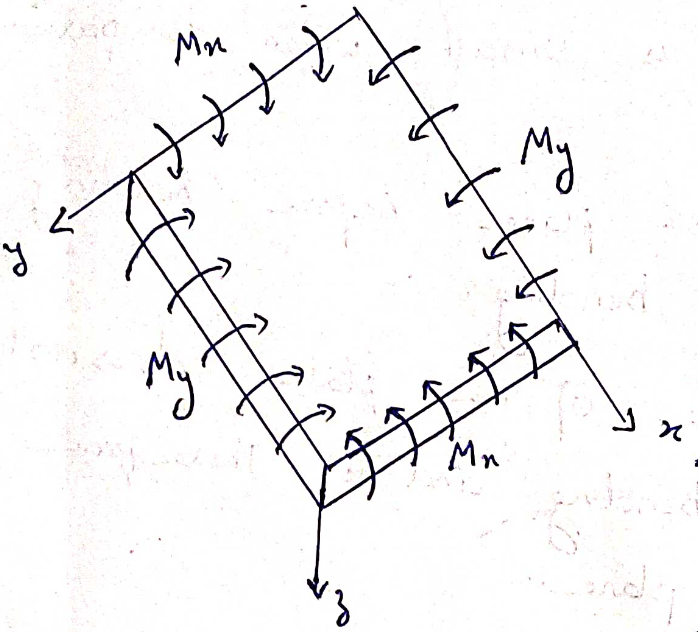
Consider an element of the Plate of side δx, δy and having a depth equal to thickness 't'. ~~δx δy~~ ρ_x, ρ_y are radii of Curvature of neutral plane in x-z and y-z plane. Positive Curvature of the plate corresponds to the positive bending moment which produce displacement in the positive direction of the z or downward direction.

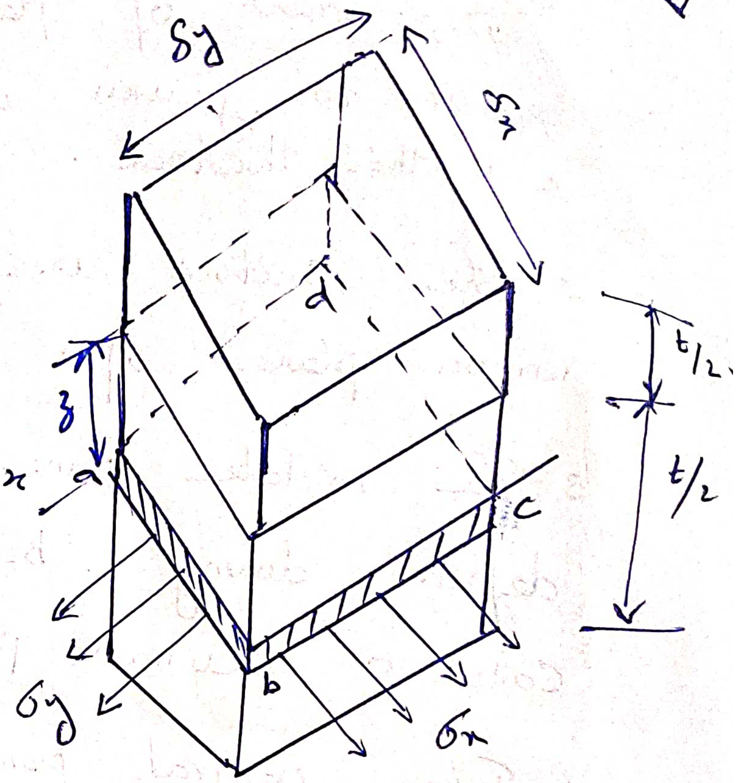
Let ε_x, ε_y be the strain in the x, & y directions respectively.

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \quad \text{—} (1)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \quad \text{—} (2)$$

Thin Rectangular Plates Subjected to bending





Let M_x, M_y be the bending moments of intensity per unit length uniformly distributed along its edges.

$M_x \Rightarrow$ Bending moment applied along the edges parallel to y-axis

$M_y \Rightarrow$ Bending moment applied along the edges parallel to x-axis

Bending moments are positive when they produce compression at the upper surface and tension at lower surface of plate.

(8) + (9) $\Rightarrow$

$$E\frac{z}{e_x}\nu + E\frac{z}{e_y} = \cancel{\nu\sigma_x} - \nu^2\sigma_y + \sigma_y - \cancel{\nu\sigma_x}$$

$$Ez\left(\frac{\nu}{e_x} + \frac{1}{e_y}\right) = \left(\sigma_y - \nu^2\sigma_y\right)$$

$$Ez\left(\frac{\nu}{e_x} + \frac{1}{e_y}\right) = \sigma_y\left(1 - \nu^2\right)$$

$$\boxed{\sigma_y = \frac{Ez}{1-\nu^2}\left(\frac{\nu}{e_x} + \frac{1}{e_y}\right)}$$

m^y

$\nu \times$ (8) $\Rightarrow$

$$\frac{zE}{e_y}\nu = \nu\sigma_y - \nu^2\sigma_x \quad \text{— (10)}$$

(8) + (10) $\Rightarrow$

$$\frac{zE}{e_x} + \frac{zE}{e_y}\nu = \sigma_x - \cancel{\nu\sigma_y} + \cancel{\nu\sigma_y} - \nu^2\sigma_x$$

$$Ez\left(\frac{1}{e_x} + \frac{\nu}{e_y}\right) = \sigma_x - \nu^2\sigma_x$$

$$Ez\left(\frac{1}{e_x} + \frac{\nu}{e_y}\right) = \sigma_x\left(1 - \nu^2\right)$$

$$\boxed{\sigma_x = \frac{Ez}{1-\nu^2}\left(\frac{1}{e_x} + \frac{\nu}{e_y}\right)}$$

σ_x & σ_y be the direct stress along x and y directions.

W.K.T bending moment eqn. is

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{\sigma}{E} = \frac{y}{R} \quad \text{ie} \quad \varepsilon = \frac{y}{R}$$

from fig.

$$\varepsilon_x = \frac{z}{\rho_x} \quad -(3)$$

$$\varepsilon_y = \frac{z}{\rho_y} \quad -(4)$$

where ρ_x & ρ_y be the radii of curvature in Neutral plane.

sub. (3) in (1) &

$$\frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{z}{\rho_x} \quad -(5)$$

$$\frac{zE}{\rho_x} = (\sigma_x - \nu\sigma_y) \quad -(6)$$

sub (4) in (2)

$$\frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{z}{\rho_y} \quad -(7)$$

$$\frac{zE}{\rho_y} = (\sigma_y - \nu\sigma_x) \quad -(8)$$

$\nu \times (6) \Rightarrow$

$$E\frac{z}{\rho_x}\nu = \nu\sigma_x - \nu^2\sigma_y \quad -(9)$$

$$= \frac{1}{3} \frac{E}{(1-\nu^2)} \left[\frac{t^3}{8} + \frac{t^3}{8} \right]$$

$$= \frac{E}{3(1-\nu^2)} \frac{\cancel{2} t^3}{\cancel{8} 4}$$

$$D = \frac{1}{12} \frac{E t^3}{1-\nu^2}$$

$$\therefore M_x = \frac{E t^3}{12(1-\nu^2)} \left[\frac{1}{\rho_x} + \frac{\nu}{\rho_y} \right]$$

$||^{ly}$

$$M_y = \int_{-t/2}^{t/2} \frac{E z}{1-\nu^2} \left(\frac{\nu}{\rho_x} + \frac{1}{\rho_y} \right) z \, dz$$

$$D = \int_{-t/2}^{t/2} \frac{E z^2}{1-\nu^2} \, dz$$

$$= \frac{E t^3}{12(1-\nu^2)}$$

$$\therefore M_y = \frac{E t^3}{12(1-\nu^2)} \left[\frac{\nu}{\rho_x} + \frac{1}{\rho_y} \right]$$

The deflection in z direction is ω.

$\therefore$ The relation b/w radius of curvature & deflection is

$$1/\rho_x = -\frac{\partial^2 \omega}{\partial x^2}$$

$$1/\rho_y = -\frac{\partial^2 \omega}{\partial y^2}$$

W.K.T

$$M_x = \int_{-t/2}^{t/2} \sigma_x \, z \, dz$$

$$M_y = \int_{-t/2}^{t/2} \sigma_y \, z \, dz$$

sub σ_x & σ_y vals.

$$M_x = \int_{-t/2}^{t/2} \frac{Ez}{1-\nu^2}\left[\frac{1}{\rho_x} + \frac{\nu}{\rho_y}\right] z \, dz$$

$$= \int_{-t/2}^{t/2} \frac{Ez^2}{1-\nu^2}\left[\frac{1}{\rho_x} + \frac{\nu}{\rho_y}\right] dz$$

D is the flexural rigidity of the plate.

$$D = \int_{-t/2}^{t/2} \frac{Ez^2}{1-\nu^2} \, dz$$

$$= \frac{E}{1-\nu^2} \int_{-t/2}^{t/2} z^2 \, dz$$

$$= \frac{E}{1-\nu^2} \left[\frac{z^3}{3}\right]_{-t/2}^{t/2}$$

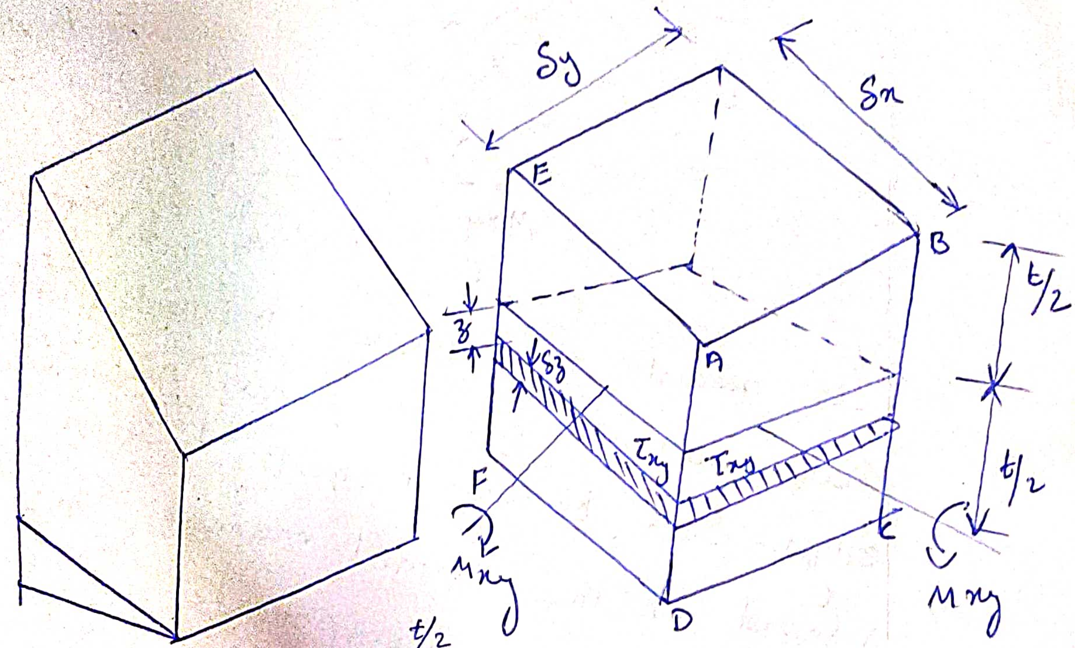
-ve Sign indicates Centre of curvature lies above the plate.

$$M_x = -\frac{Et^3}{12(1-\nu^2)}\left[\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right]$$

$$M_y = \frac{-Et^3}{12(1-\nu^2)}\left[\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right]$$

Here our aim is to relate the twisting moment M_{xy} to ω.

Consider an element of plate. The shear stresses on a lamina of the element at a distance z below the neutral plane.



$$M_{xy} = -\int_{-t/2}^{t/2} \tau_{xy}\, z\, dz$$

In terms of shear strain γ_{xy} and modulus of rigidity G

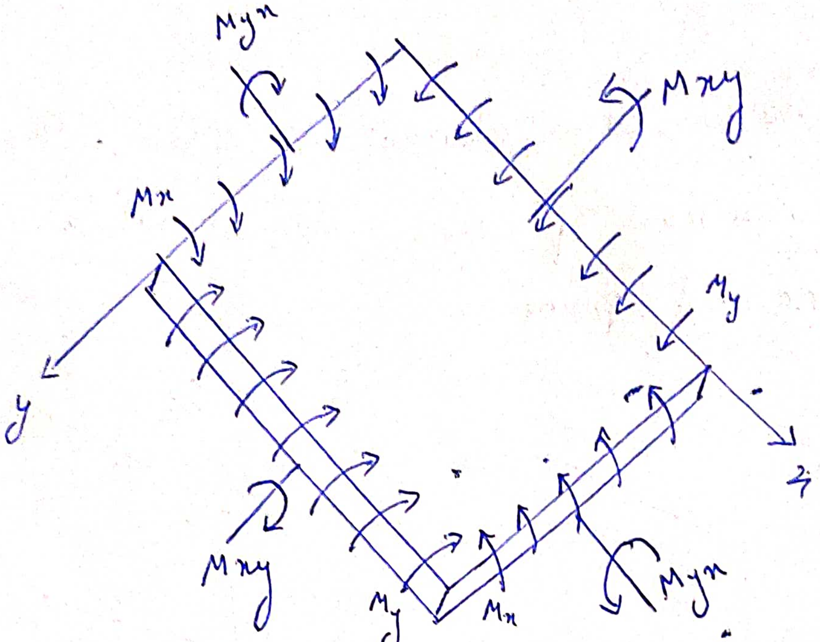
$$M_{xy} = -G\int_{-t/2}^{t/2} \gamma_{xy}\, z\, dz$$

Shear strain, $\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$

An element taken through the thickness of the plate will suffer equal rotations equal to $\frac{\partial w}{\partial x}$ & $\frac{\partial w}{\partial y}$ in xz and yz planes.

$\therefore$ Considering the rotation of such an element

Plates Subjected to Bending and Twisting



In general, the bending moments applied to the Plate will not be in planes perpendicular to its edges. Such bending moments, however may be resolved in the normal manner into tangential and perpendicular components.

M_x and M_y are the perpendicular components.

M_{xy} and M_{yx} are the tangential components.

M_{xy} is the twisting moment intensity in a vertical x plane parallel to y axis.

M_{yx} is the twisting moment intensity in a vertical y plane parallel to x axis.

Since the twisting moments are tangential moments or torque they are resisted by a System of horizontal Shear Stress τ_{xy}.

$$M_{xy} = -M_{yx}$$

$$\therefore M_{xy} = \frac{Et^3}{12(1+\nu)} \frac{\partial^2 w}{\partial x \partial y}$$

$\times$ the numerator & denominator by $(1-\nu)$

$$M_{xy} = \frac{Et^3(1-\nu)}{12(1+\nu)(1-\nu)} \frac{\partial^2 w}{\partial x \partial y}$$

$$= \frac{Et^3(1-\nu)}{12(1-\nu+\nu-\nu^2)} \frac{\partial^2 w}{\partial x \partial y}$$

$$M_{xy} = \frac{Et^3(1-\nu)}{12(1-\nu^2)} \cdot \frac{\partial^2 w}{\partial x \partial y}$$

$$\boxed{M_{xy} = D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}}$$

in xz plane, the displacement u in the x of a point at a distance z below the neutral plane is

$$u = -\frac{\partial w}{\partial x} z$$

$$v = -\frac{\partial w}{\partial y} z$$

$$\frac{\partial v}{\partial x} = -\frac{\partial^2 w}{\partial x \partial y} z$$

$$\frac{\partial u}{\partial y} = -\frac{\partial^2 w}{\partial x \partial y} z$$

$$\therefore \gamma_{xy} = -2z \frac{\partial^2 w}{\partial x \partial y}$$

$$\therefore M_{xy} = -G \int_{-t/2}^{t/2} -2z \frac{\partial^2 w}{\partial x \partial y} \times z \, dz$$

$$= G \int_{-t/2}^{t/2} 2z^2 \frac{\partial^2 w}{\partial x \partial y} \, dz$$

$$M_{xy} = 2G \frac{\partial^2 w}{\partial x \partial y} \left[\frac{z^3}{3} \right]_{-t/2}^{t/2}$$

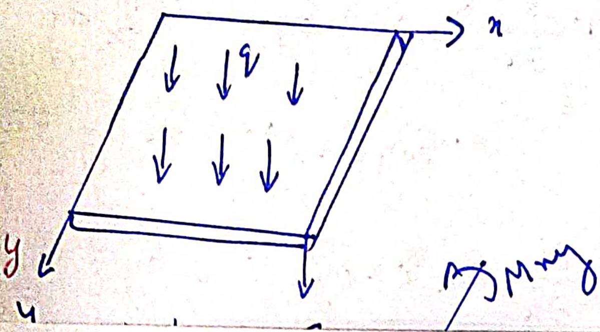
$$= \frac{2G}{3} \frac{\partial^2 w}{\partial x \partial y} \times \left[\frac{t^3}{8} + \frac{t^3}{8} \right]$$

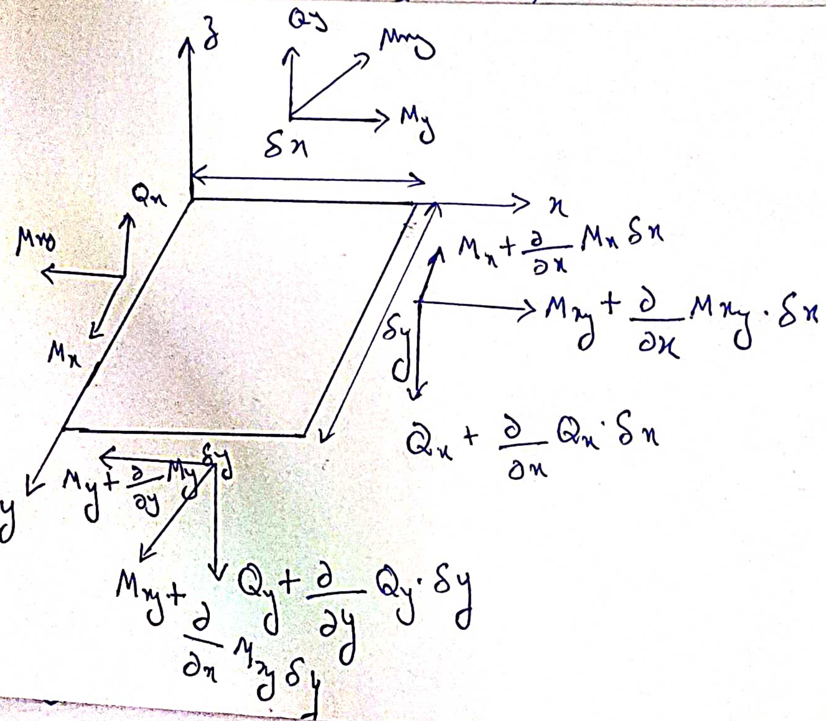
$$= \frac{\cancel{2}G}{\cancel{3}\,3} \frac{\partial^2 w}{\partial x \partial y} \times \frac{\cancel{2}t^3}{\cancel{8}\,\cancel{4}\,2}$$

$$M_{xy} = \frac{Gt^3}{6} \frac{\partial^2 w}{\partial x \partial y}$$

Modulus of rigidity, $G = \frac{E}{2(1+\nu)}$

① Thin plates Subjected to Transverse Load:





A transverse load of intensity q per unit area is applied.

The plate is subjected to bending and twisting and in addition vertical shear forces Q_x & Q_y per unit length on faces perpendicular to x & y axis respectively.

equate the forces in z direction,

$$\left(Q_x + \frac{\partial}{\partial x} Q_x \delta x\right) \delta y - Q_x \delta y + \left(Q_y + \frac{\partial}{\partial y} Q_y \delta y\right) \delta x$$

$$- Q_y \delta x + q \delta x \delta y = 0$$

$$\cancel{Q_x \delta y} + \frac{\partial}{\partial x} Q_x \delta x \delta y - \cancel{Q_x \delta y} + \cancel{Q_y \delta x} + \frac{\partial}{\partial y} Q_y \delta x \delta y$$

$$- \cancel{Q_y \delta x} + q \delta x \delta y = 0$$

$$\frac{\partial}{\partial x} Q_x \delta x \delta y + \frac{\partial}{\partial y} Q_y \delta x \delta y + q \delta x \delta y = 0 \quad \text{— (1)}$$

Moment about y axis is

$$\frac{\partial}{\partial y} M_{xy} - \frac{\partial}{\partial x} M_x + Q_x = 0 \quad \text{— (2)}$$

Moment about x axis is

$$\frac{\partial}{\partial x} M_{xy} - \frac{\partial}{\partial y} M_y + Q_y = 0 \quad \text{— (3)}$$

$$Q_x = \frac{\partial}{\partial x} M_x - \frac{\partial}{\partial y} M_{xy}$$

$$Q_y = \frac{\partial}{\partial y} M_y - \frac{\partial}{\partial x} M_{xy}$$

Sub the value of Q_x & Q_y in (1)

$$\frac{\partial}{\partial x}\left[\frac{\partial M_x}{\partial x} - \frac{\partial}{\partial y} M_{xy}\right] \delta x \delta y +$$

$$\frac{\partial}{\partial y}\left[\frac{\partial}{\partial y} M_y - \frac{\partial}{\partial x} M_{xy}\right] \delta x \delta y = - q \delta x \delta y$$

— (4)

$$-D\left[\frac{\partial^4 w}{\partial x^4} + \nu\frac{\partial^4 w}{\partial x^2 \partial y^2} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2}\right.$$

$$\left. -2\nu\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \nu\frac{\partial^4 w}{\partial x^2 \partial y^2}\right] = -q$$

$$-D\left[\frac{\partial^4 w}{\partial x^4} + \cancel{2\nu\frac{\partial^4 w}{\partial x^2 \partial y^2}} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} - \cancel{2\nu\frac{\partial^4 w}{\partial x^2 \partial y^2}}\right.$$

$$\left. + \frac{\partial^4 w}{\partial y^4}\right] = -q$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

$$\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = \frac{q}{D}$$

$$\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 = \frac{q}{D}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 w = \frac{q}{D}$$

$$\frac{\partial^2 M_x}{\partial x^2} - \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - \frac{\partial^2 M_{xy}}{\partial x \partial y} = -q$$

$$\frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q \quad \text{—(4)}$$

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)$$

$$M_{xy} = D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}$$

sub in (4)

$$\frac{\partial^2}{\partial x^2}\left[-D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)\right] + \frac{\partial^2}{\partial y^2}\left[-D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)\right] - 2\frac{\partial^2}{\partial x \partial y}\left[D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}\right] = -q$$

Wagner Beam (Diagonal Field Beam) or (Tension Field Beam)

* The ability of the thin sheet metal to carry an increasing load after it began to buckle - which in conventional structures was regarded as failure

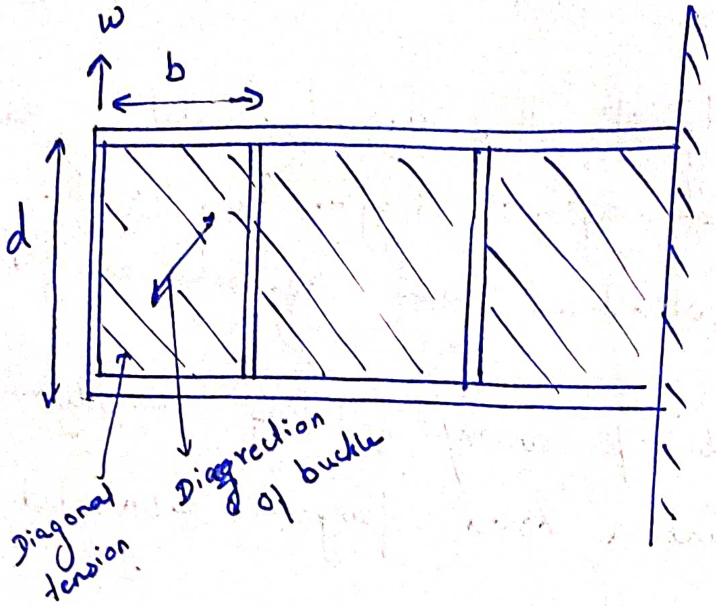
* The spars of aircraft wing usually comprise an upper and a lower flange connected by thin stiffners.

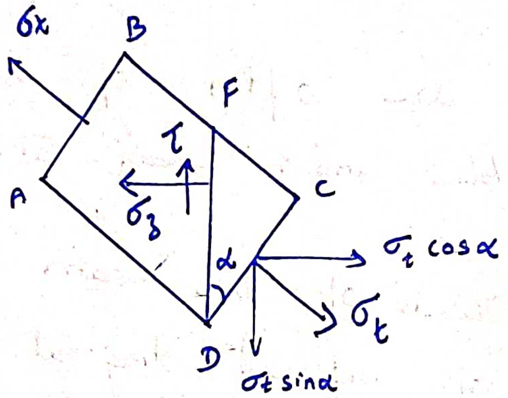
* These webs are often of such a thickness that they buckle under shear stresses.

* When the web of the beam buckles under the action of internal diagonal compressive stresses produced by shear, diagonal tension is only capable of supporting the web.

* The beam shown below has concentrated flange areas having a depth d between the flange centroids and vertical stiffeners which are placed uniformly along the length of the beam

It is assumed that the flanges resist the internal bending moment at any section of the beam, while the web of thickness t, resists the vertical shear force.





At a Section of the beam, where the Shear force is S, the Shear Stress τ is given by

$$\tau = \frac{S}{td} \Rightarrow \frac{W}{td} \quad — (1)$$

Now Consider an element ABCD of the web in a panel of the beam

The element is subjected to tensile stresses σ_t, produced by diagonal tension on the planes AB & CD, the angle of diagonal tension is α.

On the vertical plane FD is an imaginary line drawn, τ is the Shear Stress and

$\sigma_z \rightarrow$ direct stress on vertical plane, $\sigma_t \rightarrow$ tensile stress.

$$= \frac{2W}{td\, 2\sin\alpha \cancel{\cos\alpha}} \cancel{\cos\alpha} \cos\alpha$$

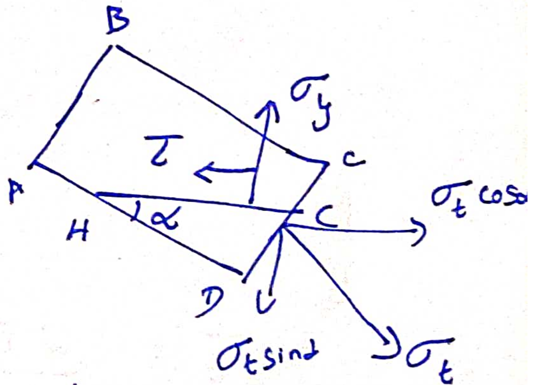
$$= \frac{W\cos\alpha}{td\,\sin\alpha}$$

$$\sigma_z = \frac{W}{td\,\tan\alpha}$$

On the horizontal plane HC is an immaginary line drawn,

$\sigma_y \Rightarrow$ direct stress

$\sigma_t \Rightarrow$ tensile stress.



resolving forces vertically

$$\sigma_y HC \times t = \sigma_t \sin\alpha \times CD \times t$$

$$\sigma_y = \sigma_t \times \frac{CD}{HC} \times \sin\alpha$$

In $\Delta HCD \Rightarrow \quad \sin\alpha = \frac{CD}{HC}$

$$\therefore \sigma_y = \sigma_t \times \sin^2\alpha$$

$$= \frac{2W}{td\,\sin 2\alpha} \times \sin^2\alpha$$

$$= \frac{\cancel{2}W}{td \times \cancel{2}\sin\cancel{\alpha}\cos\alpha} \times \sin^2\alpha$$

$$= \frac{W\sin\alpha}{td\,\cos\alpha}$$

$$\sigma_y = \frac{W\tan\alpha}{td}$$

$$= \frac{2W}{td\ 2\sin\alpha\ \cancel{\cos\alpha}}\ \cancel{\cos\alpha}\ \cos\alpha$$

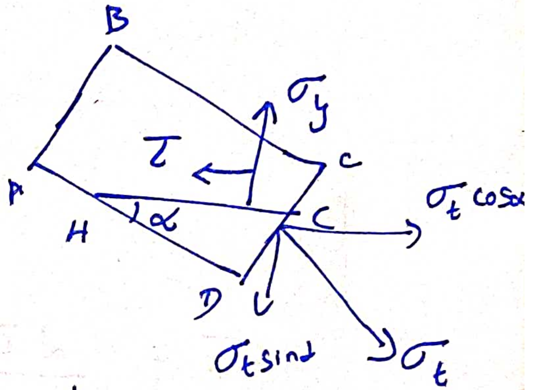
$$= \frac{W\cos\alpha}{td\ \sin\alpha}$$

$$\sigma_z = \frac{W}{td\ \tan\alpha}$$

On the horizontal plane HC is an immaginary line drawn,

$\sigma_y \Rightarrow$ direct stress

$\sigma_t \Rightarrow$ tensile stress,



resolving forces vertically.

$$\sigma_y\ HC \times t = \sigma_t \sin\alpha \times CD \times t$$

$$\sigma_y = \sigma_t \times \frac{CD}{HC} \times \sin\alpha$$

In $\Delta HCD \Rightarrow \quad \sin\alpha = \frac{CD}{HC}$

$$\therefore \sigma_y = \sigma_t \times \sin^2\alpha$$

$$= \frac{2W}{td\ \sin 2\alpha} \times \sin^2\alpha$$

$$= \frac{\cancel{2}W}{td \times \cancel{2}\cancel{\sin\alpha}\cos\alpha} \times \sin^2\alpha$$

$$= \frac{W\sin\alpha}{td\ \cos\alpha}$$

$$\sigma_y = \frac{W\tan\alpha}{td}$$

Resolving forces vertically,

$$\sigma_t \sin\alpha \times CD \times \cancel{t} = \tau\, FD \times \cancel{t}$$

$$\Delta FCD \Rightarrow \cos\alpha = \frac{CD}{FD}$$

$$\therefore \sigma_t = \tau \times \frac{FD}{CD} \times \frac{1}{\sin\alpha}$$

$$= \tau \times \frac{1}{\sin\alpha} \times \frac{1}{\cos\alpha}$$

$$= \frac{\tau}{\frac{\sin 2\alpha}{2}}$$

$$\sigma_t = \frac{2\tau}{\sin 2\alpha}$$

$$\sigma_t = \frac{2W}{\sin 2\alpha \times td} \quad \text{—— (2)}$$

Resolving forces horizontally,

$$\sigma_z\, FD \times t = \sigma_t \cos\alpha \times CD \times t$$

$$\sigma_z = \sigma_t \frac{CD}{FD} \times \cos\alpha$$

$$\Delta FCD, \quad \cos\alpha = \frac{CD}{FD}$$

$$\sigma_z = \sigma_t \cdot \cos^2\alpha$$

$$= \frac{2W}{td \sin 2\alpha} \cos^2\alpha$$

$$F_T = \frac{W z}{d} + \frac{\sigma_z t d}{2}$$

$$F_T = \frac{W z}{d} + \frac{W t \cancel{d}}{t \cancel{d} \tan\alpha \; 2}$$

$$F_T = \frac{W z}{d} + \frac{W}{2\tan\alpha}$$

equaly forces horizontally,

$$F_B + \sigma_z \times t \times d - F_T = 0$$

$$F_B = F_T - \sigma_z t d$$

$$= \frac{W z}{d} + \frac{W}{2\tan\alpha} - \frac{W}{\cancel{t d}\tan\alpha} \times t \cancel{d}$$

$$= \frac{W z}{d} + \frac{W}{2\tan\alpha} - \frac{W}{\tan\alpha}$$

$$F_B = \frac{W z}{d} - \frac{W}{2\tan\alpha}$$

$$P = \frac{W b \tan\alpha}{d}$$

If the compressive load P is high, the stiffners will buckle. Test indicate that they buckle as column of equivalent length

$$P_{cr} = \frac{\pi^2 E I}{l_e^2}$$

$$l_e = \frac{d}{\sqrt{4 - 2b/d}} \quad \text{for } b < 1.5d$$

$$l_e = d \quad \text{for } b > 1.5d$$

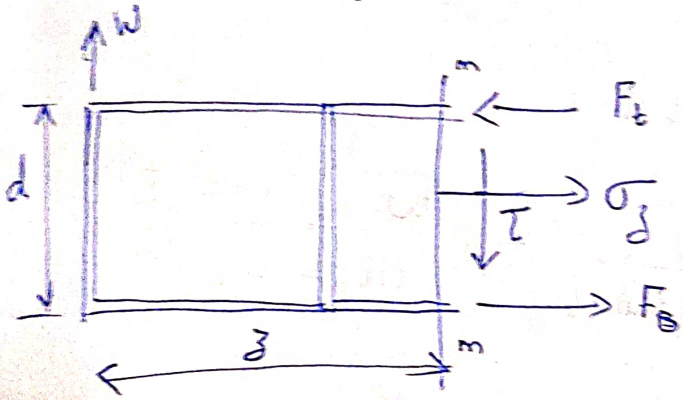
The direct stress σ_z causes compressive load in the vertical stiffeners.

$$\therefore P = \sigma_z \times t \times b$$

$$P = \frac{W \tan\alpha}{t d} \times t \times b$$

$$P = \frac{W b \tan\alpha}{d}$$

Determination of flange forces.



The direct load in the flanges are found by considering a length z of the beam.

On the plane mm there are direct and shear stresses σ_z, τ acting in the web together with F_T and F_B

taking momentum about the bottom flange,

$$Wz - F_t \times d + (\sigma_z \times t \times d)\frac{d}{2} = 0$$

$$\therefore Wz + \frac{\sigma_z t d^2}{2} = F_T d$$

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Maximum bending moment occurs at a stiffner and is given by

$$M_{max} = \frac{W b^2 \tan\alpha}{12 d}$$

Midway b/w stiffners,

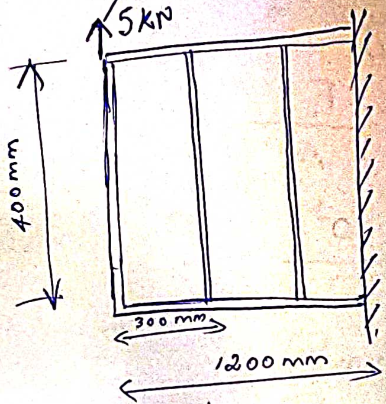
$$M_{max} = \frac{W b^2 \tan\alpha}{24 d}$$

$$\tan^4\alpha = \frac{1 + \frac{td}{2A_f}}{1 + \frac{tb}{A_s}}$$

Problem

1. A Wagner beam of length 1200mm fixed as a Cantilever is Subjected to a tip load of 5 kN. The depth of the beam is 400mm, and stiffner spacing is 300mm. The cross section areas of the flanges and stiffeners are $350 mm^2$ and $300 mm^2$ respectively. The elastic section modulus of each flange is $750 mm^3$, the thickness of web is 2mm and the 2^{nd} moment of area of stiffener about an axis in the plane of web is $2000 mm^4$. Determine the maxi Stress in a flange and also whether the stiffeners will buckle or not.

$E = 70,000 N/mm^2$



$$\tan^4\alpha = \frac{1 + \frac{td}{2A_f}}{1 + \frac{tb}{A_s}}$$

$$= \frac{1 + \frac{2 \times 400}{2 \times 350}}{1 + \frac{2 \times 300}{300}}$$

$$\tan^4\alpha = 0.714$$

$$(\tan^2\alpha)^2 = 0.714$$

$$\tan^2\alpha = 0.845$$

$$(\tan\alpha)^2 = 0.845$$

$$\tan\alpha = 0.919$$

$$\alpha = 42.6^\circ$$

Max. flange stress will occur in top flange,

$$\therefore F_T = \frac{Wz}{d} + \frac{W}{2\tan\alpha}$$

$$= \frac{5 \times 1200}{400} + \frac{5}{2\tan 42.6^\circ}$$

$$F_T = 17.720\ kN$$

Direct Stress in the top flange is.

$$\sigma_T = \frac{Load}{Area} = \frac{F_T}{A_f}$$

$$= \frac{17.720 \times 10^3}{350}$$

$$\sigma_T = 50.63\ N/mm^2$$

Bending moment

$$M_{max} = \frac{Wb^2\tan\alpha}{12d}$$

$$= \frac{5 \times 10^3 \times 300^2 \tan(42.6)}{12 \times 400}$$

$$= 8.6 \times 10^4\ Nmm$$

$$\therefore \text{Max. Compressive Stress} = \frac{8.6 \times 10^4\ Nmm}{750\ mm^3}$$

$$= 114.875\ N/mm^2$$

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$\therefore$ Total Stress in top flange $= 114.67 + 50.6$

$= 165.475 \ N/mm^2$

Compressive load in a Stiffener,

$$P = \frac{W b \tan\alpha}{d}$$

$$= \frac{5 \times 10^3 \times 300 \times \tan 42.6}{400}$$

$$= 3.448 \times 10^3 \ N$$

$$= 3.448 \ kN$$

We know,

$$P_{cr} = \frac{\pi^2 E I}{l_e^2}$$

when $b < 1.5d$

$$l_e = \frac{d}{\sqrt{4 - \frac{2b}{d}}}$$

When $b > 1.5d$

$$l_e = d$$

$$\therefore l_e = \frac{400}{\sqrt{4 - \frac{2 \times 300}{400}}}$$

$$= \frac{400}{\sqrt{4 - 1.5}} = \frac{400}{\sqrt{2.5}} = 252.988$$

$$\therefore P_{cr} = \frac{\pi^2 \times 70 \times 10^3 \times 2000}{(252.988)^2}$$

$$P_{cr} = 21.57 \ kN$$

$P < P_{cr}$ $\therefore$ safe, The stiffener will not buckle.

This given beam may be considered as two cantilevers each of length 1.2 m, built in at the midspan section carrying and carrying loads at their free ends of 5 KN. Assuming this condition the analysis of complete tension yield beam can be applied to this

$$\therefore \tan^4\alpha = \frac{1 + \frac{td}{2Af}}{1 + \frac{tb}{A_s}}$$

$\because z = 1.2\,m$
$= 1200\,mm$

$$\tan^4\alpha = \frac{1 + \frac{1.5 \times 350}{2 \times 300}}{1 + \frac{1.5 \times 300}{280}}$$

$$\alpha = 42.6^\circ$$

$$F_T = \frac{Wz}{d} + \frac{W}{2\tan\alpha}$$

$$= \frac{5 \times 1200}{\cancel{400}\,350} + \frac{5}{2\tan 42.6}$$

$$F_T = 19.9\,KN$$

2. A simply supported beam has a span of 2.4 and carries a central concentrated load of 10 kN. The flanges of the beam each have a cross sectional area of 300 mm^2, while that of the vertical web stiffener's is 280 mm^2. If the depth of the beam measured between the centeroids of area of the flanges is 350 mm and the stiffeners are symmetrically arranged about the web and spaced at 300 mm intervals, determine the maximum axial load in a flange and compressive load in the stiffener. It may be assumed that the beam web, of thickness 1.5 mm, is capable of resisting diagonal tension only.

$t = 1.5 \text{ mm}$

$d = 350 \text{ mm}$

$b = 300 \text{ mm}$

$A_f = 300 \text{ mm}^2$

$A_s = 280 \text{ mm}^2$

length = 2.4 m

Central load = 10 kN

Compressive load in a stiffness,

$$P = \frac{W b \tan\alpha}{d}$$

$$= \frac{5 \times 300 \times \tan 42.6}{350}$$

$$P = 3.9\ kN$$

3. A thin square plate of side a and thickness t is simply supported along each edge, and has a slight initial curvature giving an initial deflected shape.

$$w_0 = \delta \sin\frac{\pi x}{a} \sin\frac{\pi y}{a}$$

If the plate is subjected to a uniform compressive stress σ in the x direction find an expression for the elastic deflection w normal to the plate. Show also the deflection at mid point of the plate can be presented in the form of a Southwell plot and illustrate your answer with suitable sketch.

Where

$$w_1 = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{a}$$

where,

$$B_{mn} = \frac{A_{mn} N_x}{(\pi^2 D/a^2)\left[m + (n^2a^2/mb^2)\right]^2 - N_x}$$

here $m = n = 1$, $a = b$ and $N_x = \sigma t$, $A = \delta$

$$B_{mn} = \frac{\delta \sigma t}{(\pi^2 D/a^2)\left[1 + \left(\frac{a^2}{a^2}\right)\right]^2 - \sigma t}$$

$$= \frac{\delta \sigma t}{(\pi^2 D/a^2)(1+1)^2 - \sigma t}$$

$$= \frac{\delta \sigma t}{(4\pi^2 D/a^2) - \sigma t}$$

where
$m \to$ No. of half waves in x direction
& $n \to$ No. of half waves in y direction

A_{mn} & B_{mn} are unknown coefficients that must satisfy the above differential eqns.

~~When $\sigma t \to 4\pi^2 D/a^2$, $w \to \infty$ and $\sigma t \to N_{xcr}$ the buckling load of the plate may be written as W_c.~~

Sub B_{mn} in w,

ie $$w_1 = \frac{\delta \sigma t}{(4\pi^2 D/a^2) - \sigma t} \sin \frac{\pi x}{a} \sin \frac{\pi y}{a}$$

the deflection at centre of the plate where $x = a/2$, $y = b/2$, then

$$W_c = \frac{\delta \sigma t}{(4\pi^2 D/a^2) - \sigma t} \; \sin\frac{\pi \times a}{2 \times a} \; \sin\frac{\pi \times a}{2 \times a}$$

$$W_c = \frac{\delta \sigma t}{(4\pi^2 D/a^2) - \sigma t} \times 1 \times 1$$

ie $$W_c = \frac{\delta \sigma t}{(4\pi^2 D/a^2) - \sigma t}$$

when $\sigma t \to 4\pi^2 D/a^2$, $w \to \infty$ and $\sigma t \to N_{x\,CR}$ the buckling load of the plate may be written as

$$W_c = \frac{\delta \sigma t}{N_{x\,CR} - \sigma t}$$

$$W_c = \frac{\dfrac{\delta \sigma t}{N_{x\,CR}}}{1 - \dfrac{\sigma t}{N_{x\,CR}}}$$

ie The graph can be drawn for W_c against σt which will be a straight line of slope $N_{x\,CR}$ and intercepts at δ

ie. Southwell plot.

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Unit - II

Unsymmetrical Bending

Moment of Inertia

The moment of a force also called the 1st moment of ~~fixed~~ force about any point is the product of the force and the ⊥r distance between them. If this 1st moment of force is again multiplied by the ⊥r distance it is called the 2nd moment of force. If instead of force, the area of the object is considered, then it is called 2nd moment of area or moment of Inertia.

Principal plane and Stress.

The plane carrying the maximum normal stress is called the major principal plane and the corresponding stress is major principal stress. The plane carrying the minimum normal stress is known as minor principal plane and the corresponding stress is known as minor principal stress.

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) \cancel{\pm} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

σ_1, σ_2 are major & minor principal stress.

Principal moment of Inertia.

If the two axes about which the product of inertia is found such that the product of inertia becomes zero, the two axes are then called principal axes.

The moment of inertia about the principal axis is called the principal moment of inertia

Let us consider a 2D fig. where c.g is the centre of gravity, the elementary axis will pass through c.g. Let u-u and v-v be the principal axes.

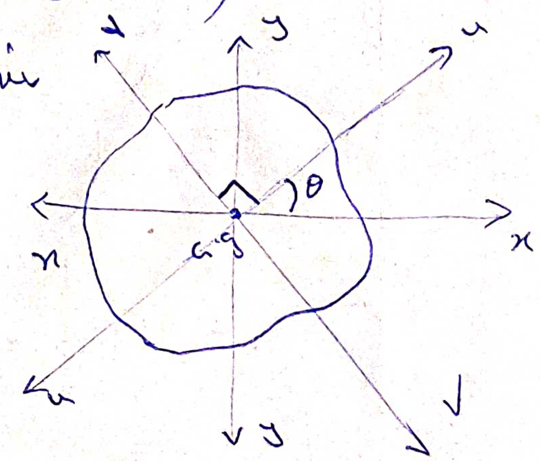
u-u → major principal axis
v-v → minor principal axis
I_{uu} → maximum moment of inertia
I_{vv} – minimum moment of Inertia

$$I_{uu} = \frac{I_{xx}+I_{yy}}{2} + \sqrt{\left(\frac{I_{yy}-I_{xx}}{2}\right)^2 + I_{xy}^2}$$

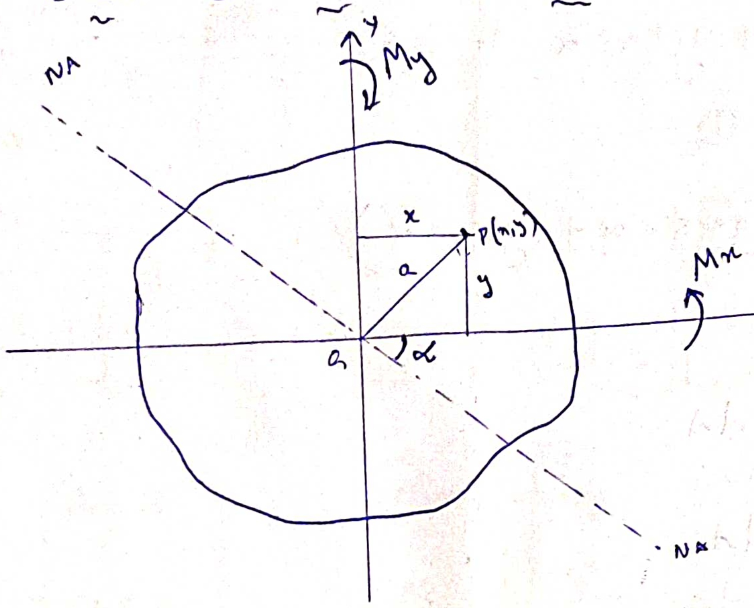
$$I_{vv} = \frac{I_{xx}+I_{yy}}{2} - \sqrt{\left(\frac{I_{yy}-I_{xx}}{2}\right)^2 + I_{xy}^2}$$

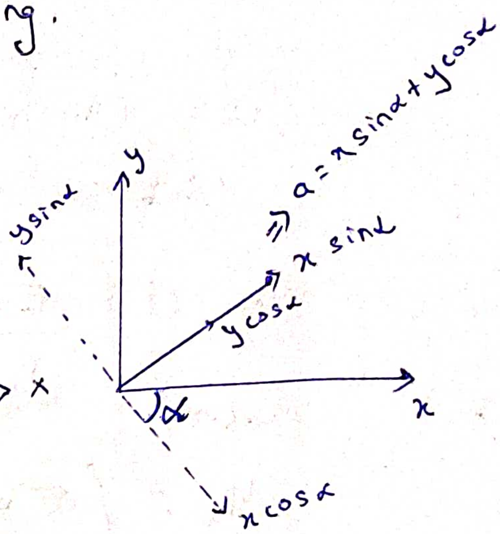
Location of principal axis

$$\tan 2\theta = \frac{2 I_{xy}}{I_{yy} - I_{xx}}$$



Direct Stress Distribution due to bending.





Consider the cross section of beam as in fig.
Let M_x & M_y be the bending moment about some axis of the cross section. Let G be the centroid of the section. Then XX & YY be the mutually $\perp^r$ axis passing through the centroid. Consider neutral axis is passing through the centroid. Let 'p' be a point at x and y distance from the axis. 'a' be the distance between neutral axis and point 'p'. Let σ be the stress of the beam is bend to a radii of curvature about N.A, at this section,

$$\frac{\sigma}{\varepsilon} = E$$

w.k,

$$\frac{M}{I} = \frac{\sigma}{a} = \frac{E}{r}$$

$$\sigma = E\left(\frac{a}{r}\right) \quad \text{——} \quad (1)$$

where $\varepsilon = \dfrac{a}{r}$

The beam is subjected to pure bending, therefore

$$\int \sigma \, dA = 0$$

$a = x \sin\alpha + y\cos\alpha$ — ②

Sub ② in ①

ie $\sigma = \frac{E}{Y}(x\sin\alpha + y\cos\alpha)$

W.K.T

$M_x = \int \sigma y \, dA$

$M_y = \int \sigma x \, dA$

$I_{xx} = \int y^2 \, dA$

$I_{yy} = \int x^2 \, dA$

$I_{xy} = \int xy \, dA$

$M_x = \frac{E}{Y}\int (x\sin\alpha + y\cos\alpha) y \, dA$

$= \frac{E}{Y}\int xy \sin\alpha \, dA + \frac{E}{Y}\int y^2 \cos\alpha \, dA$

$M_x = \frac{E}{Y}\left[I_{xy}\sin\alpha + I_{xx}\cos\alpha\right]$ — ③

$M_y = \int \sigma x \, dA$

$= \frac{E}{Y}\int (x\sin\alpha + y\cos\alpha) x \, dA$

$= \frac{E}{Y}\int x^2 \sin\alpha \, dA + \frac{E}{Y}\int xy\cos\alpha \, dA$

$$M_y = \frac{E}{r}\left[I_{yy}\sin\alpha + I_{xy}\cos\alpha \right]$$

$$\begin{bmatrix} M_x \\ M_y \end{bmatrix} = \frac{E}{r}\begin{bmatrix} I_{xy} & I_{xx} \\ I_{yy} & I_{xy} \end{bmatrix}\begin{bmatrix} \sin\alpha \\ \cos\alpha \end{bmatrix}$$

$$\frac{E}{r}\begin{bmatrix} \sin\alpha \\ \cos\alpha \end{bmatrix} = \begin{bmatrix} M_x \\ M_y \end{bmatrix}\begin{bmatrix} I_{xy} & I_{xx} \\ I_{yy} & I_{xy} \end{bmatrix}^{-1}$$

$$= \frac{\begin{bmatrix} I_{xy} & -I_{xx} \\ -I_{yy} & I_{xy} \end{bmatrix}\begin{bmatrix} M_x \\ M_y \end{bmatrix}}{\begin{vmatrix} \cancel{I_{xy}^2 - I_{xx} I_{yy}} & \\ I_{xy} & I_{xx} \\ I_{yy} & I_{xy} \end{vmatrix}}$$

$$= \frac{\begin{bmatrix} I_{xy} & -I_{xx} \\ -I_{yy} & I_{xy} \end{bmatrix}\begin{bmatrix} M_x \\ M_y \end{bmatrix}}{I_{xy}^2 - I_{xx} I_{yy}}$$

$$\frac{E}{r}\begin{bmatrix} \sin\alpha \\ \cos\alpha \end{bmatrix} = \frac{1}{I_{xy}^2 - I_{xx} I_{yy}}\begin{bmatrix} I_{xy} M_x - I_{xx} M_y \\ -I_{yy} M_x + I_{xy} M_y \end{bmatrix}$$

$$\frac{E}{r}\sin\alpha = \frac{I_{xy}M_x - I_{xx}M_y}{I_{xy}^2 - I_{xx}I_{yy}}$$

$$\frac{E}{r}\cos\alpha = \frac{-I_{yy}M_x + I_{xy}M_y}{I_{xy}^2 - I_{xx}I_{yy}}$$

$$\frac{E}{r}\sin\alpha \cdot x = \frac{I_{xx}M_y - I_{xy}M_x}{I_{xx}I_{yy} - I_{xy}^2}\,x$$

$$\frac{E}{r}\cos\alpha \cdot y = \frac{-I_{xy}M_y + I_{yy}M_x}{I_{xx}I_{yy} - I_{xy}^2}\,y$$

$$\frac{E}{r}\sin\alpha \cdot x + \frac{E}{r}\cos\alpha \cdot y = \frac{I_{xx}M_y - I_{xy}M_x}{I_{xx}I_{yy} - I_{xy}^2}\,x + \frac{-I_{xy}M_y + I_{yy}M_x}{I_{xx}I_{yy} - I_{xy}^2}\,y$$

$$\boxed{\therefore\ \sigma = \left[\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right]x + \left[\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right]y}$$

at neutral axis, $\sigma = 0$

$$\therefore \left[\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] x + \left[\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] y = 0$$

$$\left[\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] y = - \left[\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] x$$

$$\frac{y}{x} = \frac{M_x I_{xy} - M_y I_{xx}}{\cancel{I_{xx} I_{yy} - I_{xy}^2}} \times \frac{\cancel{I_{xx} I_{yy} - I_{xy}^2}}{-M_x I_{yy} + M_y I_{xy}}$$

$$= \frac{M_x I_{xy} - M_y I_{xx}}{-M_x I_{yy} + M_y I_{xy}}$$

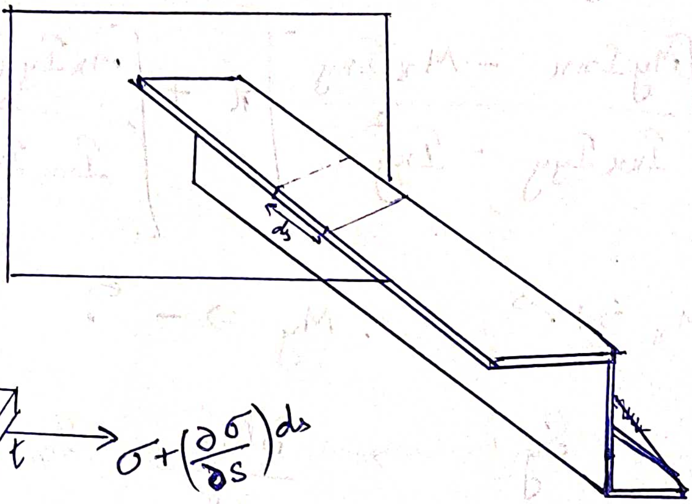
$$\boxed{\tan\alpha = \frac{M_y I_{xx} - M_x I_{xy}}{M_x I_{yy} - M_y I_{xy}}}$$

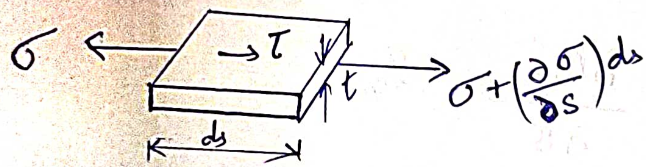
Shear flow in Open Section:

Shear flow is defined as the shear force resistance per unit length. It is denoted by q.

$$q = \frac{\text{Shear force}}{\text{length}} \quad N/m$$

Consider a cantilever beam of any loading system in to it.





$$dA = t\cdot ds$$

$$\text{Shear Force} = \tau \times dA$$

$$F = \tau \times t \times ds$$

$$\text{Shear flow}, q = \frac{F}{ds} = \frac{\tau\cdot t\cdot \cancel{ds}}{\cancel{ds}}$$

$$q = \tau\cdot t$$

Static equilibrium eqn. $\Sigma F = 0$

$$-\cancel{\sigma dA} + \cancel{\sigma dA} + \left(\frac{\partial\sigma}{\partial s}\right) ds\cdot dA + \tau\cdot dA = 0$$

$$\left(\frac{\partial\sigma}{\partial s}\right) ds\cdot dA = -\tau \times t\cdot ds$$

$$\int \frac{\partial \sigma}{\partial s} ds \cdot dA = -\tau \cdot t \cdot ds .$$

$$\int \frac{\partial \sigma}{\partial s} dA = -\tau \cdot t$$

$$q = \tau \cdot t$$

ie $-q = \int \frac{\partial \sigma}{\partial s} dA$.

$$\sigma = \left[\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] x + \left[\frac{M_x I_{yy} - M_y I_{xx}}{I_{xx} I_{yy} - I_{xy}^2}\right] y$$

$M_x \Rightarrow +$, $M_y \Rightarrow -$

$$\frac{\partial M_x}{\partial s} = S_y \quad , \quad \frac{\partial M_y}{\partial s} = S_x$$

S_x and S_y are the shear loads acting at shear centre such that there is no twisting.

$$\therefore -q = \int_{dA} \left[\frac{\frac{\partial M_y}{\partial s} I_{xx} - \frac{\partial M_x}{\partial s} I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] x \, dA + \int_{dA} \left[\frac{\frac{\partial M_x}{\partial s} I_{yy} - \frac{\partial M_y}{\partial s} I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] y \, dA$$

$$-q = \frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \int_A x \, dA + \frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \int_A y \, dA$$

$$-q = \frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \int x \cdot t\, ds + \frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \int y \cdot t \cdot ds .$$

$$\boxed{q = \frac{S_y I_{xy} - S_x I_{xx}}{I_{xx} I_{yy} - I_{xy}^2} \int x \cdot t\, ds + \frac{S_x I_{xy} - S_y I_{yy}}{I_{xx} I_{yy} - I_{xy}^2} \int y \cdot t \cdot ds}$$

If the orgin of the Shear flow is at starting of the open Section, then $q = 0$.

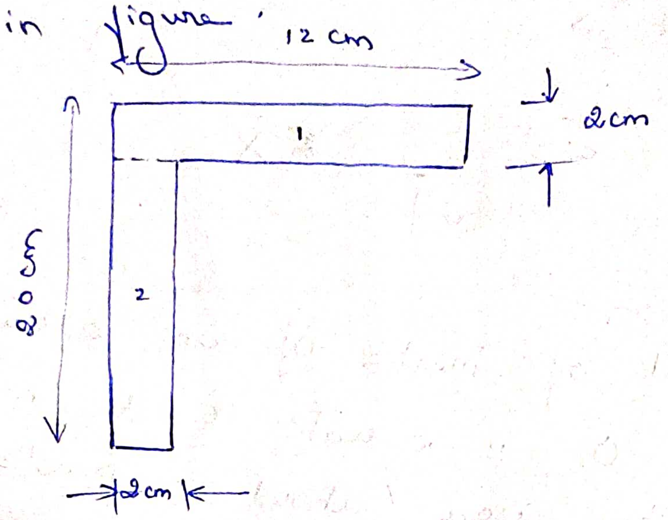
for Symmetric Sections, $I_{xy} = 0$

$$\therefore q = \left[\frac{-S_x \cancel{I_{xx}}}{\cancel{I_{xx}} I_{yy}} \int x \cdot t \cdot ds \right] + \left[\frac{-S_y \cancel{I_{yy}}}{I_{xx} \cancel{I_{yy}}} \int y\, t\, ds \right]$$

$$\boxed{q = -\frac{S_x}{I_{yy}} \int_0^s x \cdot t \cdot ds - \frac{S_y}{I_{xx}} \int_0^s y\, t \cdot ds}$$

Problem:

1. Find the principal moment of inertia and directions of principal axis for the angle section shown in figure.



Section	Area cm^2	x cm	y cm	h for I_{yy} $(x-\bar{x})$	h for I_{xx} $(y-\bar{y})$
1	24	6	19	3	6
2	36	1	9	-2	-4

$$I_{uu} = \left(\frac{I_{xx}+I_{yy}}{2}\right) + \sqrt{\left(\frac{I_{yy}-I_{xx}}{2}\right)^2 + I_{xy}^2}$$

$$I_{vv} = \left(\frac{I_{xx}+I_{yy}}{2}\right) - \sqrt{\left(\frac{I_{yy}-I_{xx}}{2}\right)^2 + I_{xy}^2}$$

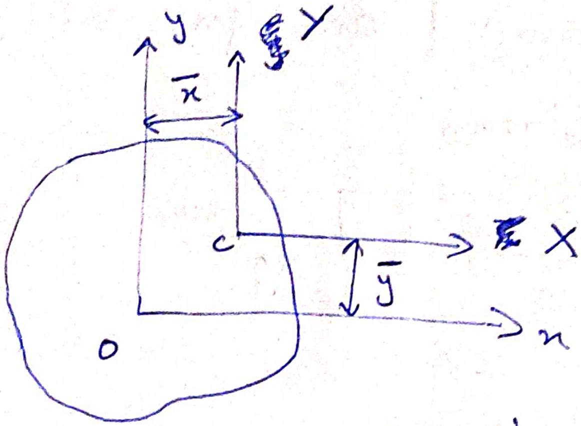
$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$\bar{x} = \frac{24 \times 6 + 36 \times 1}{24+36}$$

$$= 3 \text{ cm}$$

Parallel axis theorem.



The moment of inertia of an area about an axis through O is equal to the moment of inertia of the area about a parallel axis through the centroid C + the area multiplied by the square of distance between the axis

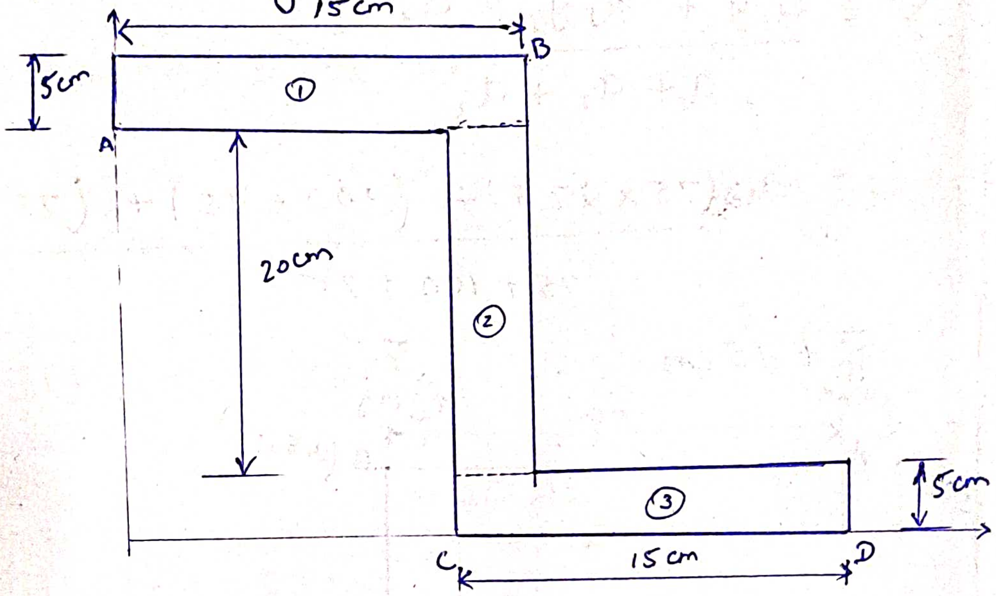
$$I_{XX} = I_{xx} + A\bar{y}^2$$

$$I_{YY} = I_{yy} + A\bar{x}^2$$

Problems

1. Find the bending stress values of the points A, B, C & D for the section shown in fig.

$M_x = 10$ kNm & $M_y = 10$ kNm



$$\sigma_b = \left[\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] x + \left[\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] y$$

Section	Area cm^2	x	y	h for I_{xx} $y-\bar{y}$	h for I_{yy} $x-\bar{x}$
1	75	7.5	27.5	12.5	-5
2	100	12.5	15	0	0
3	75	17.5	2.5	-12.5	5

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

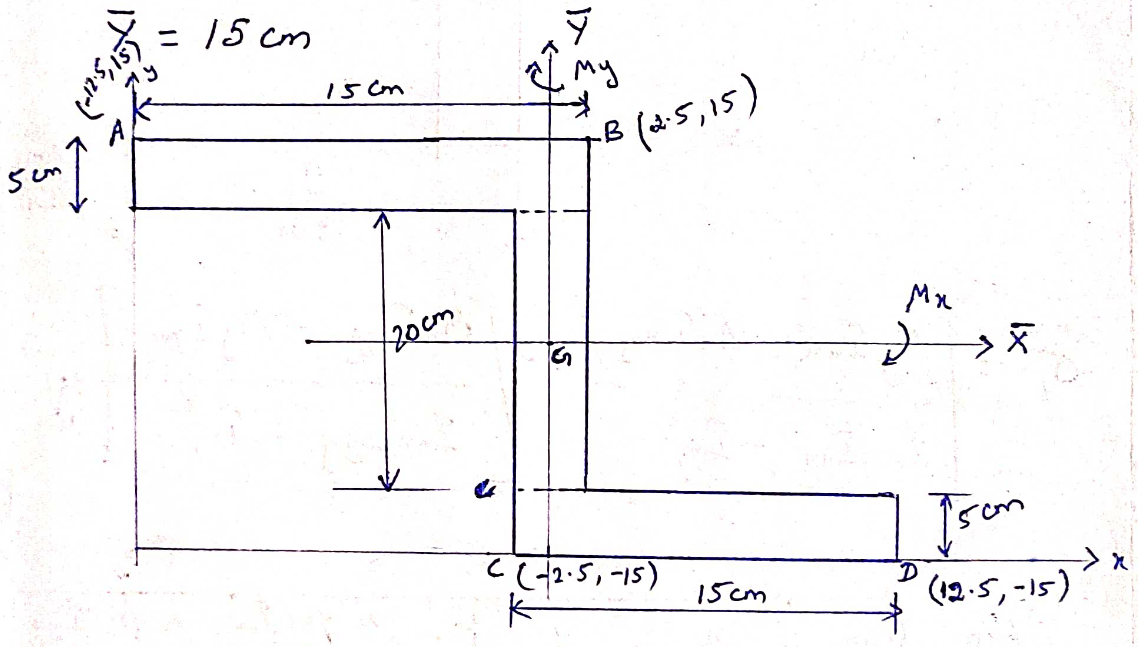
$$\bar{X} = \frac{(75 \times 7.5) + (100 \times 12.5) + (75 \times 17.5)}{75 + 100 + 75}$$

$$= 12.5 \text{ cm}$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(75 \times 27.5) + (100 \times 15) + (75 \times 2.5)}{75 + 100 + 75}$$

$\bar{Y} = 15$ cm



$$I_{xx_1} = \frac{bd^3}{12} = \frac{15 \times 5^3}{12} = 156.25 \text{ cm}^4$$

$$I_{xx_2} = \frac{bd^3}{12} = \frac{5 \times 20^3}{12} = 3333.33 \text{ cm}^4$$

$$I_{xx_3} = \frac{15 \times 5^3}{12} = 156.25 \text{ cm}^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{5 \times 15^3}{12} = 1406.25 \text{ cm}^4$$

$$I_{yy_2} = \frac{20 \times 5^3}{12} = 208.333 \text{ cm}^4$$

$$I_{yy_3} = \frac{5 \times 15^3}{12} = 1406.25 \text{ cm}^4$$

$$I_{xx} = \left[I_{xx_1} + A_1 h_1^2\right] + \left[I_{xx_2} + A_2 h_2^2\right] + \left[I_{xx_3} + A_3 h_3^2\right]$$

$$= (156.25 + (75 \times 12.5^2)) + (3333.33 + (100 \times 0^2))$$

$$+ (156.25 + (75 \times -12.5^2))$$

$$I_{xx} = 27083.33 \text{ cm}^4$$

$$I_{yy} = \left[I_{yy_1} + A_1 h_1^2\right] + \left[I_{yy_2} + A_2 h_2^2\right] + \left[I_{yy_3} + A_3 h_3^2\right]$$

$$= \left[1406.25 + (75 \times -5^2)\right] + \left[208.333 + (100 \times 0^2)\right] +$$

$$\left[1406.25 + (75 \times 5^2)\right].$$

$$I_{yy} = 6770.833 \text{ cm}^4$$

$$I_{x_1y_1} = A_1 (x_1 - \bar{x})(y_1 - \bar{y})$$

$$= 75 \times -5 \times 12.5$$

$$= -4687.5 \text{ cm}^4$$

$$I_{x_2y_2} = A_2 (x_2 - \bar{x})(y_2 - \bar{y})$$

$$= 100 \times 0 \times 0$$

$$= 0$$

$$I_{x_3y_3} = A_3 (x_3 - \bar{x})(y_3 - \bar{y})$$

$$= 75 \times 5 \times -12.5$$

$$= -4687.5 \text{ cm}^4$$

$$I_{xy} = -9375 \text{ cm}^4$$

$$\sigma_b = \left[\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] x + \left[\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] y$$

$$M_x = 10 \text{ kNm}$$
$$= 10 \times 10^2 \text{ kN cm}$$

$$M_y = 10 \text{ kNm}$$
$$= 10 \times 10^2 \text{ kN cm}$$

$$\sigma = \left[\frac{10 \times 10^2 \times 27083.33 - 10 \times 10^2 \times (-9375)}{27083.33 \times 6770.833 - 9375^2}\right] x$$

$$+ \left[\frac{10 \times 10^2 \times 6770.833 - 10 \times 10^2 \times (-9375)}{27083.33 \times 6770.833 - 9375^2}\right] y$$

$$= \cancel{\left[\frac{364583330}{1833679 14.9}\right]} x + \cancel{\left[\frac{16145833}{18336791 4.9}\right]} y$$

$$\cancel{\sigma = 0.1988 x + 0.0880 y}$$

$$\sigma = 0.3818 x + 0.169 y \quad \text{kN/cm}^2$$

$$\sigma = 381.8 x + 169.09 y \quad \text{N/cm}^2$$

A is at $(-12.5, 15)$ from Centroid

$$\therefore \sigma_A = 381.8 \times -12.5 + 169.09 \times 15$$

$$\sigma_A = -2236.15 \ N/cm^2$$

Negative sign indicates compression.

σ_B is at $(2.5, 15)$

$$\sigma_B = 381.8 \times 2.5 + 169.09 \times 15$$

$$= 3490.85 \ N/cm^2$$

C is at $(-2.5, -15)$

$$\sigma_c = (381.8 \times -2.5) + (169.09 \times -15)$$

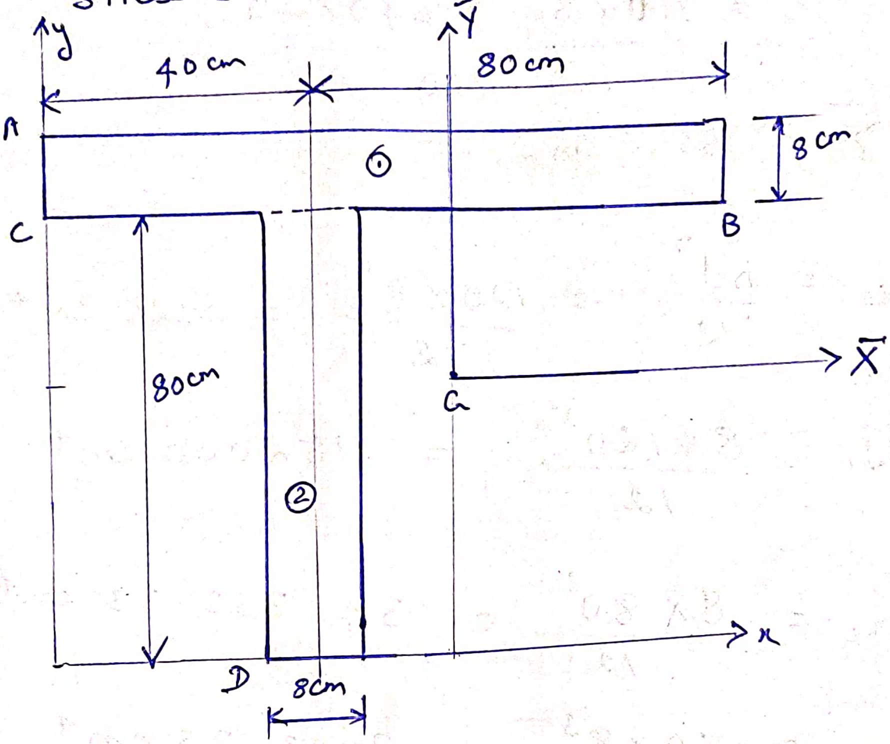
$$= -3490.85 \ N/cm^2$$

D is at $(12.5, -15)$

$$\sigma_D = 381.8 \times 12.5 + 169.09 \times (-15)$$

$$\sigma_D = 2236.15 \ N/cm^2$$

2. The Section Shown in fig. is Subjected to a bending moment of $M_x = 30\,kNm$. Determine the bending stresses at the corner points A, B, C & D.



Section	Area cm^2	x cm	y cm	h for I_{xx} $y-\bar{Y}$	h for I_{yy} $x-\bar{X}$
①	960	60	84	17.6	8
②	640	40	40	-26.4	-12

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{960 \times 60 + 640 \times 40}{1600} = 52\,cm$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{960 \times 84 + 640 \times 40}{1600}$$

$$\bar{Y} = 66.4 \text{ cm}$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{120 \times 8^3}{12} = 5120 \text{ cm}^4$$

$$I_{yy_1} = \frac{8 \times 120^3}{12} = 1152000 \text{ cm}^4$$

$$I_{xx_2} = \frac{8 \times 80^3}{12} = 341333.33 \text{ cm}^4$$

$$I_{yy_2} = \frac{80 \times 8^3}{12} = 3413.33 \text{ cm}^4$$

$$I_{XX} = (I_{xx_1} + A_1 h_1^2) + (I_{xx_2} + A_1 h_2^2)$$

$$= \left[5120 + 960 \times (17.6)^2\right] + \left[341333.3 + 640 \times (26.4)^2\right]$$

$$I_{xx} = 302489.6 + 787387.73$$

$$I_{xx} = 1089877.33 \text{ cm}^4$$

$$I_{YY} = (I_{yy_1} + A_1 h_1^2) + (I_{yy_2} + A_2 h_2^2)$$

$$= \left[1152000 + 960 \times 8^2\right] + \left[3413.33 + 640 \times 12^2\right]$$

$$I_{yy} = 1213440 + 95573.33$$

$$= 1309013.33 \text{ cm}^4$$

$$I_{xy} = A_1(x_1 - \bar{x})(y_1 - \bar{y}) + A_2(x_2 - \bar{x})(y_2 - \bar{y})$$

$$= 960(8)(17.6) + 640 \times -12 \times -26.4$$

$$= 135168 + 202752$$

$$= 337920 \text{ cm}^4$$

$$\sigma = \left[\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] x + \left[\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] y$$

$$M_x = 30 \text{ kNm}$$

$$= 30 \times 10^2 \text{ kN cm}$$

$$M_y = 0$$

$$\sigma = \left[\frac{-30 \times 10^2 \times 337920}{(1089877.33 \times 1309013.33) - 337920^2}\right] x$$

$$+ \left[\frac{30 \times 10^2 \times 1309013.33}{(1089877.33 \times 1309013.33) - 337920^2}\right] y$$

$$\sigma = -7.7240 \times 10^{-4} x + 2.9 \times 10^{-3} y$$

$$\sigma = -0.000772\, x + 0.0029\, y \quad kN/cm^2$$

$A(-52,\ 21.6)$

$B(68,\ 13.6)$

$C(-52,\ 13.6)$

$D(-16,\ -66.4)$

at A,

$$\sigma_A = -0.000772 \times (-52) + 0.0029 \times 21.6$$

$$\sigma_A = 0.1027 \ kN/cm^2$$

at B,

$$\sigma_B = -0.000772\,(68) + 0.00299 \times 13.6$$

$$\sigma_B = -0.01292 \ kN/cm^2$$

(-ve sign indicats compression)

at C

$$\sigma_C = -0.000772\,(-52) + 0.0029 \times 13.6$$

$$= 0.079 \ kN/cm^2$$

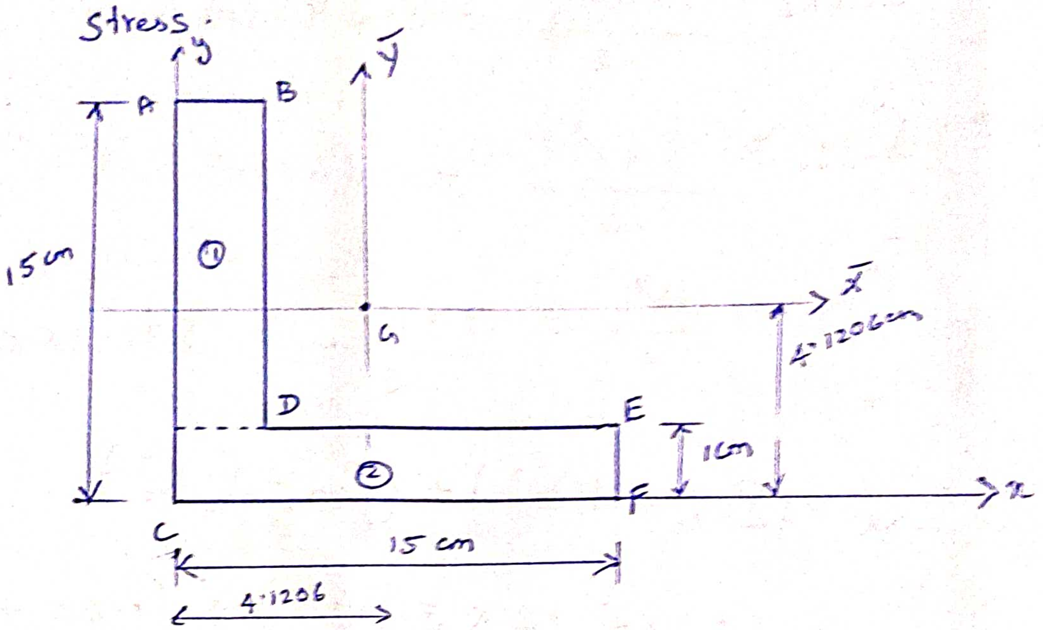
at D

$$\sigma_D = -0.000772\,(-16) + 0.0029\,(-66.4)$$

$$= -0.186 \ kN/cm^2$$

(-ve sign indicats compression.

3. An angle section shown in fig. is subjected to $M_x = 20$ kNm & $M_y = 15$ kNm. Find the maximum bending stress.



Section	Area cm^2	x cm	y cm	h for I_{xx} $y-\bar{y}$ cm	h for I_{yy} $x-\bar{x}$ cm
1	14	0.5	8	3.879	−3.6206
2	15	7.5	0.5	−3.6206	3.379

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{14 \times 0.5 + 15 \times 7.5}{14+15} = 4.1206 \text{ cm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{14 \times 8 + 15 \times 0.5}{14+15}$$

$$= 4.1206 \text{ Cm}$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{1 \times 14^3}{12} = 228.66\ cm^4$$

$$I_{xx_2} = \frac{15 \times 1^3}{12} = 1.25\ cm^4$$

$$I_{yy_1} = \frac{14 \times 1^3}{12} = 1.66\ cm^4$$

$$I_{yy_2} = \frac{db^3}{12} = \frac{1 \times 15^3}{12} = 281.25\ cm^4$$

$$I_{xx} = \left(I_{xx_1} + A_1 h_1^2\right) + \left(I_{xx_2} + A_2 h_2^2\right)$$

$$= \left(228 + 14 \times 3.879^2\right) + \left(1.25 + 15 \times (-3.6206)^2\right)$$

$$I_{xx} = 637.17\ cm^4$$

$$I_{yy} = \left(I_{yy_1} + A_1 h_1^2\right) + \left(I_{yy_2} + A_2 h_2^2\right)$$

$$= \left[1.66 + 14(-3.6206)^2\right] + \left[281.25 + 15(3.379)^2\right]$$

$$I_{yy} = 637.202\ cm^4$$

$$I_{xy} = A_1(x_1 - \bar{x})(y_1 - \bar{y}) + A_2(x_2 - \bar{x})(y_2 - \bar{y})$$

$$= 14(0.5 - 4.1206)(8 - 4.1206) +$$

$$15(-3.6206) \times (3.379)$$

$$I_{xy} = -380.15\ cm^4$$

$\sigma_A = (6.566 \times -4.1206) + (7.054 \times 10.879)$

$= 49.720 \ kN/cm^2$

$\sigma_B = (6.566 \times -3.1206) + (7.054 \times 10.879)$

$= 56.28 \ kN/cm^2$

$\sigma_C = (6.566 \times -4.1206) + (7.054 \times -4.1206)$

$= -56.08 \ kN/cm^2$

$\sigma_D = (6.566 \times -3.1206) + (7.054 \times -3.1206)$

$= -42.4 \ kN/cm^2$

$\sigma_E = (6.566 \times 10.879) + (7.054 \times -3.1206)$

$= 49.36 \ kN/cm^2$

$\sigma_F = (6.566 \times 10.88) + (7.054 \times -4.12)$

$= 42.31$

$\therefore$ at pt. B tensile Stress is maximum and at point C, Compressive Stress is maximum.

$$\sigma = \left[\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] x + \left[\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right] y$$

$M_x = 20 \times 10^2$ kN cm

$M_y = 15 \times 10^2$ kN cm

$$\sigma = \left[\frac{15 \times 10^2 \times 637.17 - 20 \times 10^2 \times -380.15}{(637.17 \times 637.202) - (-380.15)^2}\right] x$$

$$+ \left[\frac{20 \times 10^2 \times 637.202 - 15 \times 10^2 (-380.15)}{(637.17 \times 637.202) - (-380.15)^2}\right] y$$

$\sigma = 6.566 x + 7.054 y$

A $(-4.1206,\ 10.879)$

B $(-3.1206,\ 10.879)$

C $(-4.1206,\ -4.1206)$

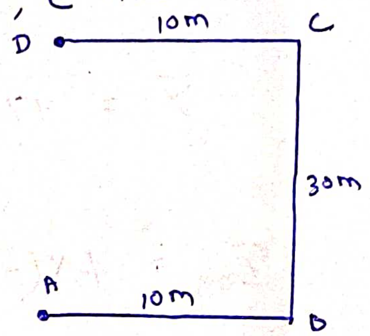
D $(-3.1206,\ -3.1206)$

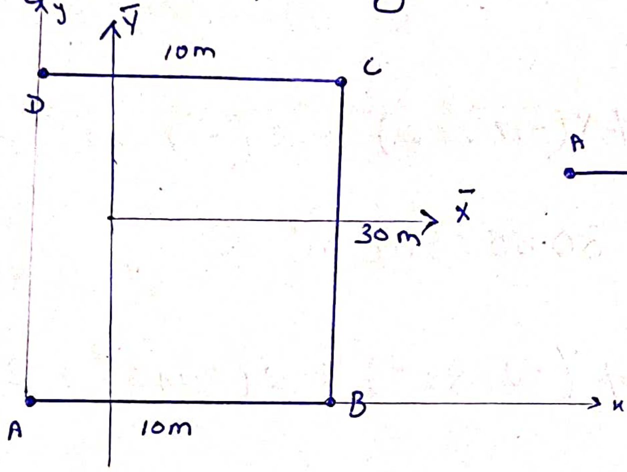
E $(10.879,\ -3.1206)$

F $(10.879,\ -4.1206)$

Shear flow in open Section - Problems.

1. Calculate the Shear flow and shear Center for the Section shown in fig. The section is subjected to a Shear force of 1 kN in vertical and horizontal directions, lumped areas at A, B, C & D are $4m^2$, $2m^2$, $2m^2$ & $6m^2$ respectively.





Member	Area m^2	x m	y m	h for I_{xx} $y-\bar{y}$	h for I_{yy} $x-\bar{x}$
A	4	0	0	-17.142	-2.857
B	2	10	0	-17.142	7.143
C	2	10	30	12.858	7.143
D	6	0	30	12.858	-2.857

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4}{a_1 + a_2 + a_3 + a_4}$$

$$= \frac{(4 \times 0) + (2 \times 10) + (2 \times 10) + (6 \times 0)}{4 + 2 + 2 + 6}$$

$$\bar{X} = 2.857\ m$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + a_4 y_4}{a_1 + a_2 + a_3 + a_4}$$

$$= \frac{(4\times 0) + (2\times 0) + (2\times 30) + (6\times 30)}{4+2+2+6}$$

$$\bar{Y} = 17.14\ m$$

$$I_{xx} = 4\times(-17.142)^2 + 2(-17.142)^2 + 2(12.858)^2 + 6(12.858)^2$$

$$= 3085.906\ m^4$$

$$I_{yy} = 4(-2.857)^2 + 2(7.143)^2 + 2(7.143)^2 + 6(-2.857)^2$$

$$= 285.7\ m^4$$

$$I_{xy} = 4(-2.857)(-17.142) + 2(7.143)(-17.142) +$$

$$2(7.143)(12.858) + 6(-2.857)(12.858)$$

$$I_{xy} = -85.71\ m^4$$

$$q = \left[\frac{S_y I_{xy} - S_x I_{xx}}{I_{xx} I_{yy} - I_{xy}^2}\right] A_i x_i + \left[\frac{S_x I_{xy} - S_y I_{yy}}{I_{xx} I_{yy} - I_{xy}^2}\right] A_i y_i$$

Case (i)

$S_x = 0, \quad S_y = 1\,kN = 1000\,N$

$$q = \left[\frac{1000 \times (-85.71) - 0}{3085.906 \times 285.7 - (85.7)^2}\right] A_i x_i +$$

$$\left[\frac{0 - (1000 \times 285.7)}{(3085.906 \times 285.7) - (85.7)^2}\right] A_i y_i$$

$$q = -0.098\, A_i x_i - 0.326\, A_i y_i$$

$$q_{AB} = (-0.098 \times 4 \times -2.857) - (0.326 \times 4 \times -17.142)$$

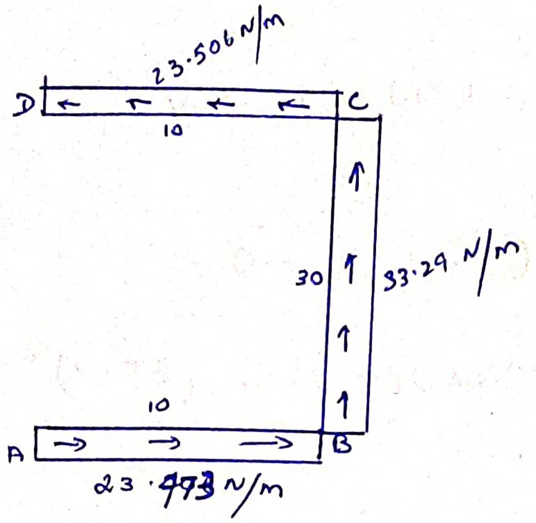
$$= 23.473\ N/m$$

$$q_{BC} = q_{AB} + \left[-0.098 \times 2 \times 7.143 - (0.326 \times 2 \times -17.142)\right]$$

$$= 33.29\ N/m$$

$$q_{CD} = q_{BC} + \left[-0.098 \times 2 \times 7.143 - (0.326 \times 2 \times 12.858)\right]$$

$$= 23.506\ N/m$$



Shear flow = $\frac{\text{Shear force}}{\text{unit length}}$

take moment about D,

$$S_y \times e_x = (23.473 \times 10 \times 30) + (33.29 \times 30 \times 10)$$

$$1000 \times e_x = 17028.9$$

$$e_x = 17.03 \text{ m}$$

Case (ii)

$$S_x = 1000 \text{ N}, \quad S_y = 0$$

$$q = \left[\frac{0 - 1000 \times 3085}{3085 \times 285.7 - (85.7)^2}\right] A_i x_i + \left[\frac{1000 \times (-85.7) - 0}{3085 \times 285.7 - (85.7)^2}\right] A_i y_i$$

$$q = -3.529 A_i x_i \cancel{+} - 0.098 A_i y_i$$

$$q_{AB} = (-3.529 \times 4 \times -2.857) - (0.098 \times 4 \times (-17.142))$$

$$= 47.049 \text{ N/m}$$

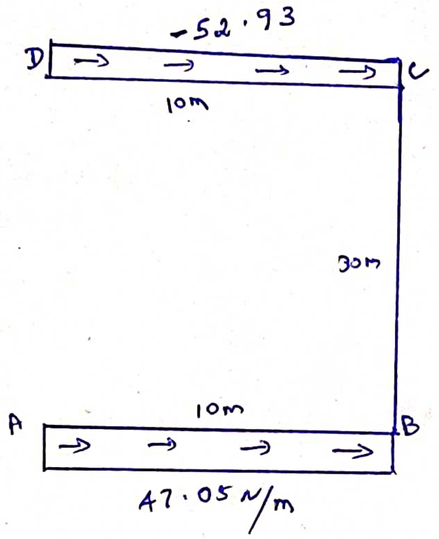
$$q_{BC} = q_{AB} + \left[-3.529 \times 2 \times 7.143 - 0.098 \times 2 \times (-17.142)\right]$$

$$= -0.0064$$

$$q_{BC} \simeq 0$$

$$q_{CD} = q_{BC} + \left[-3.529 \times 2 \times 7.143 - 0.098 \times 2 \times 12.858\right]$$

$$= -52.93$$



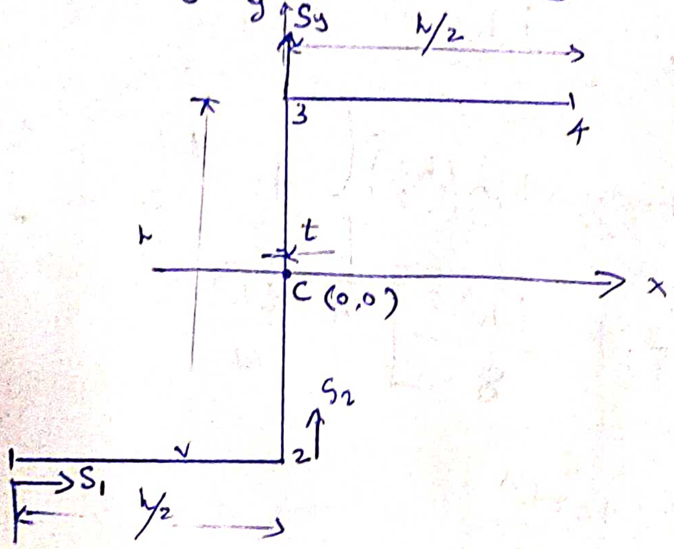
taking moment about D

$$S_x \times e_y = 47.05 \times 10 \times 30 - 0$$

$$e_y = 14.115\,m$$

Thin Walled Section - Problems:

1. Determine the shear flow distribution in the thin walled Z-section shown in fig. which is subjected to a shear load S_y applied through the shear centre of the section.



$S_y \Rightarrow$ given

$\therefore S_x = 0$

$$q = \frac{S_y I_{xy} - S_x I_{xx}}{I_{xx} I_{yy} - I_{xy}^2} \int_0^s x\, t\, ds + \frac{S_x I_{xy} - S_y I_{yy}}{I_{xx} I_{yy} - I_{xy}^2} \int_0^s y \cdot t\, ds.$$

$S_x = 0$

$$\therefore q = \frac{S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \int x\, t\, ds - \frac{S_y I_{yy}}{I_{xx} I_{yy} - I_{xy}^2} \int y \cdot t\, ds$$

$$q = \frac{S_y}{I_{xx} I_{yy} - I_{xy}^2} \left[I_{xy} \int x\, t\, ds - I_{yy} \int y\, t\, ds. \right]$$

2

I_{xx} & B

Section	x	y
1-2	$-h/4$	$-h/2$
2-3	0	0
3-4	$h/4$	$h/2$

$$I_{xx} = 2\left\{\left[\frac{h/2 \times t^3}{12}\right] + h/2 \times t \times \left(-h/2\right)^2\right\} + \frac{th^3}{12} + t \times h \times 0$$

$$= 2\left[\frac{ht^3}{24} + \frac{th^3}{8}\right] + \frac{th^3}{12}$$

Neglect higher orders of t

$$I_{xx} = \cancel{2} \times \frac{th^3}{\cancel{8}4} + \frac{th^3}{12}$$

$$= \frac{th^3}{4}\left[1 + \frac{1}{3}\right]$$

$$\boxed{I_{xx} = \frac{th^3}{3}}$$

$$I_{yy} = 2\left[\frac{t(h/2)^3}{12} + t \times h/2 \times (-h/4)^2\right] + \frac{ht^3}{12} + h \times t \times 0$$

Neglect higher orders.

$$= 2\left[\frac{th^3}{96} + \frac{th^3}{32}\right]$$

$\frac{12}{96}$

$$= \frac{th^3}{48} + \frac{th^3}{16}$$

$\frac{16}{3} \over 96$

$$= \frac{th^3}{16}\left[1 + \frac{1}{3}\right] = \frac{th^3}{\cancel{16}4} \times \frac{\cancel{4}}{3}$$

$$\boxed{\therefore I_{yy} = \frac{th^3}{12}}$$

$$I_{xy} = 2\left[\frac{h}{2}\times t \times \frac{h}{4} \times \frac{h}{2}\right] + h \times t \times 0$$

$$= \cancel{2}\left[\frac{th^3}{\cancel{16}\,8}\right]$$

$$\boxed{I_{xy} = \frac{th^3}{8}}$$

$$\therefore q = \frac{S_y}{\left(\frac{th^3}{3}\times\frac{th^3}{12}\right)-\left(\frac{th^3}{8}\right)^2}\left[\frac{th^3}{8}\int x \cdot t\, ds - \frac{th^3}{12}\int y\, t\, ds\right]$$

$$= \frac{S_y}{\frac{t^2h^6}{36} - \frac{t^2h^6}{64}}\left[\frac{th^3}{8}\int x\, t\, ds - \frac{th^3}{12}\int y\, t\, ds\right]$$

$$= \frac{S_y}{\frac{64t^2h^6 - 36t^2h^6}{2304}}\left[\frac{th^3}{8}\int x\, t\, ds - \frac{th^3}{12}\int y\, t\, ds\right]$$

$$= \frac{S_y \times 2304}{28\cancel{t^2}\cancel{h^6}h^3}\left[\frac{\cancel{t^2}\cancel{h^3}\times}{8}\int x\, ds\right] - \frac{2304 S_y}{28\cancel{t^2}\cancel{h^6}h^3}\left[\frac{\cancel{t^2}\cancel{h^3}}{12}\int y\, ds\right]$$

$$q = \frac{10.28\, S_y}{h^3}\int_0^s x\, ds - \frac{6.85\, S_y}{h^3}\int_0^s y\, ds$$

Consider bottom flange, 4

$$x = -h/2 + S_1, \quad y = -h/2$$

$$\therefore q_{12} = \frac{10.28 S_y}{h^3} \int_0^S \left(-h/2 + S_1\right) ds_1 - \frac{6.85 S_y}{h^3} \int_0^S \left(-h/2\right) ds_1$$

$$= \frac{10.28 S_y}{h^3} \left[-\frac{h}{2} \times S_1 + \frac{S_1^2}{2} \right]_0^{S_1} - \frac{6.85 S_y}{h^3} \left[-\frac{h}{2} S_1 \right]$$

$$= \frac{S_y}{h^3} \left\{ 10.28 \left[-\frac{h S_1}{2} + \frac{S_1^2}{2} \right] + 6.85 \frac{h S_1}{2} \right\}$$

$$= \frac{S_y}{h^3} \left[-5.14 h S_1 + 5.14 S_1^2 + 3.42 h S_1 \right]_0^{S_1}$$

$$q_{12} = \frac{S_y}{h^3} \left[-1.72 h S_1 + 5.14 S_1^2 \right]_0^{S_1}$$

at point 1, $S_1 = 0$

$$\therefore q_1 = 0$$

at point 2, $S_1 = h/2$

$$\therefore q_2 = \frac{S_y}{h^3} \left[-1.72 \times h \times h/2 + 5.14 \times \frac{h^2}{4} \right] + q_1$$

$$q_2 = \frac{S_y}{h^3}\left[-1.72 \times \frac{h^2}{2} + \frac{5.14\,h^2}{4}\right] + 0$$

$$= -\frac{0.86\,S_y}{h} + \frac{1.285\,S_y}{h}$$

$$\boxed{q_2 = \frac{0.425\,S_y}{h}}$$

at web 2-3

$x = 0,\quad y = -h/2 + S_2$

$$q_{23} = \frac{-6.85}{h^3} S_y \int_0^{S_2} \left(-h/2 + S_2\right) dS_2 + q_2$$

$$= \frac{-6.85}{h^3} S_y \left[-\frac{h\,S_2}{2} + \frac{S_2^2}{2}\right]_0^{S_2} + q_2$$

$$= \frac{-6.85}{h^3} S_y \times \frac{-h\,S_2}{2} - \frac{6.85}{h^3} \times \frac{S_2^2 \times S_y}{2} + q_2$$

$$q_{23} = \frac{3.425\,S_y \times S_2}{h^2} - \frac{3.425\,S_2^2 \times S_y}{h^3} + q_2$$

at Pt. 2, $S_2 = 0$

$$q_2 = \frac{0.425\,S_y}{h}$$

at Pt. 3

$$S_2 = h$$

$$q_3 = \frac{3.425 \times S_y \times \cancel{h}}{h^2} - \frac{3.425 \times S_y \times \cancel{h^2}}{h^3} + q_2$$

$$= \frac{3.425\, S_y}{h} - \frac{3.425}{h} S_y + \frac{0.425\, S_y}{h}$$

$$\boxed{q_3 = \frac{0.425\, S_y}{h}}$$

at mid of web, $S_2 = h/2$

$$q_{mid\ web} = \frac{3.425}{h^2} \times \frac{h}{2} - \frac{3.425}{h^3} \times \frac{h^2}{4} + q_2$$

$$= \frac{3.425\, S_y}{2h} - \frac{3.425\, S_y}{4h} + q_2$$

$$= \frac{S_y}{2h}\left[3.425 - \frac{3.425}{2}\right] + q_2$$

$$= \frac{1.7125\, S_y}{2h} + \frac{0.425\, S_y}{h}$$

$$= \frac{0.856\, S_y}{h} + \frac{0.425\, S_y}{h}$$

$$q_{mid} = \frac{S_y}{h} \times 1.281$$

$$q_{mid} = \frac{1.281\, S_y}{h}$$

at pt. 1
$q_{12} = 0$

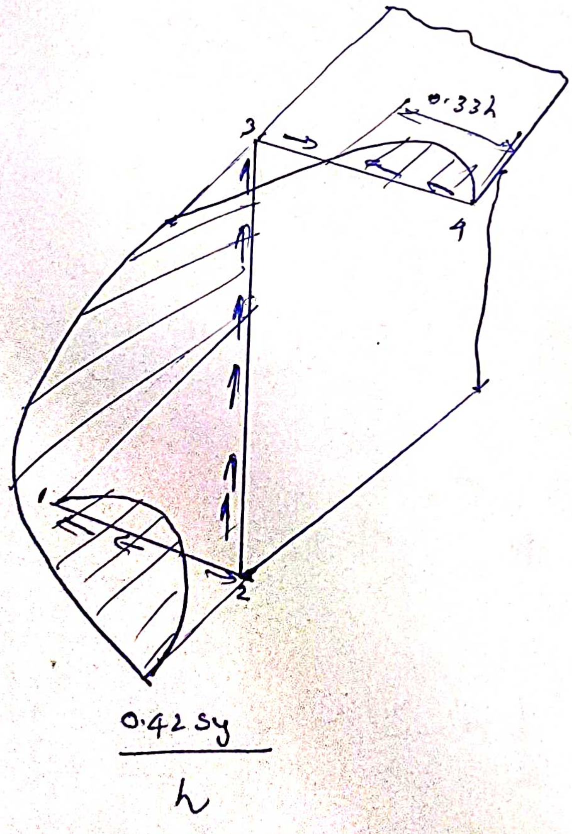
$$\frac{S_y}{h^3}\left[-1.72\,h S_1 + 5.14 S_1^2\right] = 0$$

$$5.14 S_1^2 - 1.72\,h S_1 = 0$$

$$\therefore S_1 = \frac{+1.72\,h \pm \sqrt{(1.72h)^2 - 4 \times 5.14 \times 0}}{2 \times 5.14}$$

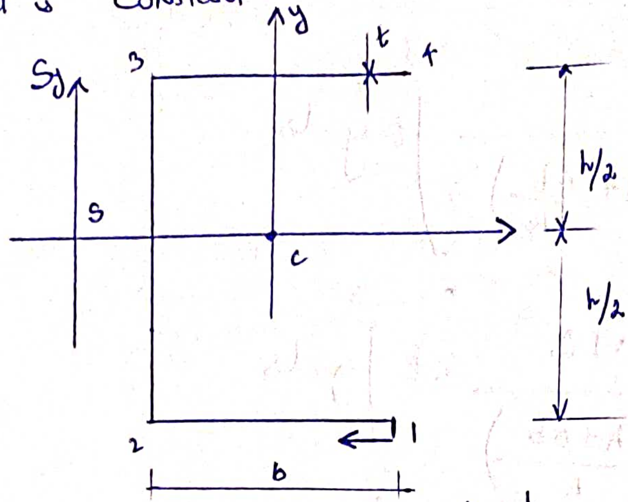
$$S_1 = \frac{1.72\,h + 1.72\,h}{2 \times 5.14} \Big/ \frac{1.72h - 1.72h}{2 \times 5.14}$$

$$\therefore S_1 = \cancel{0} \; 0.334\,h$$



①

1. Calculate the position of shear centre of the thin walled channel section shown in fig. The thickness 't' of the wall is constant.



This cross section is symmetry about x-axis

∴ Shear centre lies on the x axis ∴ $I_{xy} = 0$

$$q_s = -\left[\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right]\int_0^s t x\, ds - \left[\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right]\int t y\, ds$$

$$S_x = 0, \quad I_{xy} = 0$$

$$\therefore q_s = \frac{-S_y \cancel{I_{yy}}}{I_{xx} \cancel{I_{yy}}}\int t y\, ds$$

$$= -\frac{S_y}{I_{xx}}\int t y\, ds$$

$$I_{xx} = 2\left[\frac{bt^3}{12} + bt \times \left(\frac{h}{2}\right)^2\right] + \frac{t \times h^3}{12}$$

$$= 2\left[\frac{bt^3}{12} + \frac{bth^2}{4}\right] + \frac{th^3}{12}$$

Neglecting higher orders of t

$$I_{xx} = \frac{bth^2}{2} + \frac{th^3}{12}$$

$$\boxed{I_{xx} = \frac{th^3}{12}\left[1 + \frac{6b}{h}\right]}$$

$$q_s = \frac{-S_y}{\frac{th^3}{12}\left(1+\frac{6b}{h}\right)} \int_0^s t\,y\,ds$$

$$= \frac{-S_y \times 12}{\cancel{t}h^{\cancel{3}\,2}\left(\frac{h+6b}{\cancel{h}}\right)} \times \cancel{t} \int_0^s y\,ds$$

$$= \frac{-S_y \times 12}{h^2(6b+h)} \int_0^s y\,ds.$$

bottom flange: $y = -h/2$

$$q_{12} = \frac{-S_y \times 12}{h^2(6b+h)} \int_0^{s_1} -h/2 \, ds.$$

$$= \frac{-\cancel{12}S_y}{h^2(6b+h)} \times \frac{-1}{2}\left[h\,s_1\right]_0^{s_1}$$

$$= \frac{6S_y}{h^{\cancel{2}}(6b+h)} \cancel{h}\left[s_1\right]_0^{s_1}$$

$$q_{12} = \frac{6S_y}{h(6b+h)}\left[s_1\right]_0^{s_1}$$

at pt. 1, $s_1 = 0$

$\therefore q_1 = 0$

at pt. 2, $s_1 = b$

$$\therefore q_2 = \frac{6S_y b}{h(6b+h)}$$

In web, $y = -h/_2 + S_2$

$$q_{23} = \frac{-S_y \times 12}{h^2(6b+h)} \int_0^s \left(-h/_2 + S_2\right) ds_2 + q_2$$

$$= \frac{-S_y \times 12}{h^2(6b+h)} \left[\frac{-h S_2}{2} + \frac{S_2^2}{2}\right]_0^s + q_2$$

$$q_{23} = \frac{-6 S_y}{h^2(6b+h)} \left[S_2^2 - h S_2\right]_0^{S_2} + q_2$$

at pt. 3,

$S_2 = h$

$$\therefore q_{23} = \frac{-6 S_y}{h^2(6b+h)} \left[h^2 - h^2\right] + q_2$$

$$q_3 = \frac{6 S_y b}{h(6b+h)}$$

~~take moment about mid~~

$$\cancel{S_y \times e_x = 2\left[\frac{6 S_y b}{h(6b+h)}\right]}$$

$$\cancel{e_x = \frac{12 b}{6h\left(\frac{6b}{h}+ h\right)}}$$

In web, $y = -h/2 + S_2$

$$q_{23} = \frac{-S_y \times 12}{h^2(6b+h)} \int_0^{s} \left(-h/2 + S_2\right) ds_2 + q_2$$

$$= \frac{-S_y \times 12}{h^2(6b+h)} \left[-\frac{h S_2}{2} + \frac{S_2^2}{2} \right]_0^{s} + q_2$$

$$q_{23} = \frac{-6 S_y}{h^2(6b+h)} \left[S_2^2 - h S_2 \right]_0^{S_2} + q_2$$

at pt. 3,

$S_2 = h$

$$\therefore q_{23} = \frac{-6 S_y}{h^2(6b+h)} \left[h^2 - h^2 \right] + q_2$$

$$q_3 = \frac{6 S_y b}{h(6b+h)}$$

~~take moment about mid~~

$$\cancel{S_y \times e_x = 2\left[\frac{6 S_y b}{h(6b+h)}\right]}$$

$$\cancel{e_x = \frac{12b}{6h}}$$

take moment w.r to x axis

$$S_y \times e_x = 2\left[\int_0^b q_{12}\, ds_1 \left(h/2\right)\right]$$

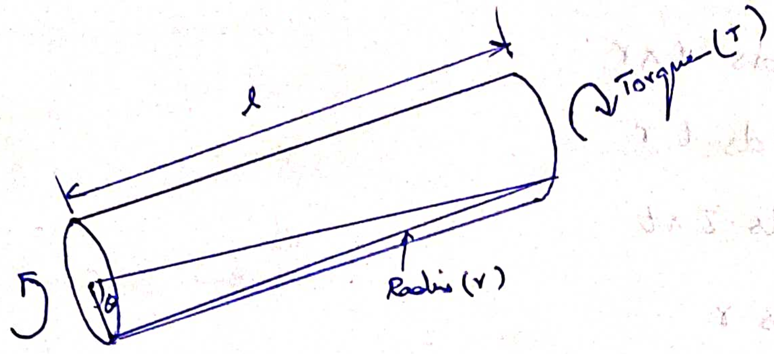
$$= \cancel{2}\int_0^b \frac{6 S_y S_1}{\cancel{h}(6b+h)} \times ds_1 \times \cancel{h}/\cancel{2}$$

$$= \frac{6 S_y}{(6b+h)}\left[\frac{S_1^2}{2}\right]_0^b$$

$$\cancel{S_y}\, e_x = \frac{3 \cancel{S_y}\, b^2}{(6b+h)}$$

$$\boxed{e_x = \frac{3b^2}{6b+h}}$$

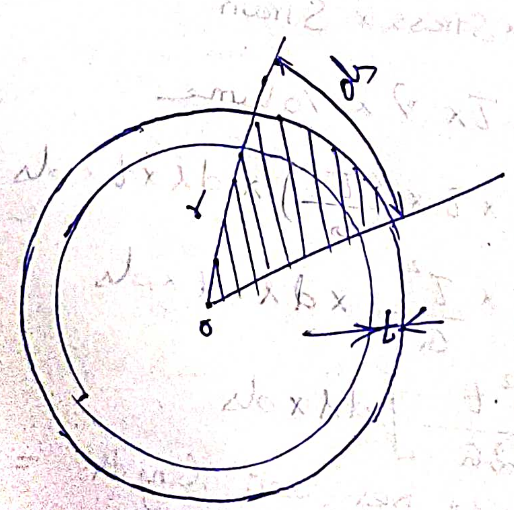
Shear flow in Closed Section due to Torsion (using Bredt Batho formula)



If the loads are applied away from the Shear Center, torsion besides bending also occurs. Therefore the beam is subjected to stress due to torsion and bending.

Torsion causes a twisting stress 'τ' also called shear stress and a rotation called shear strain γ. Torsional moment increases linearly as shear strain in it increases.

Let us consider torque is acting on the elemental area of length 'ds' and radius 'r'



$q = \tau \times t$

Torsional moment,

$$dT = dF \cdot r$$

$$dT = \tau \times ds \cdot t \times r$$

$$\int dT = \int \tau \times ds \cdot t \cdot r$$

$$= \int r \cdot ds \cdot \tau \times t$$

$$\int dT = \int q \cdot ds \cdot r$$

$$T = q \int r \cdot ds$$

In Δ^{le},

$$dA = \frac{1}{2} \times ds \times r$$

$$r \cdot ds = 2\, dA$$

$$T = q \int 2\, dA$$

$$\boxed{T = 2Aq}$$ → Bredt batho formula

$$q = \frac{T}{2A}$$

Sheer stress,

$$\tau = \frac{q}{t}$$

$$\tau = \frac{T}{2At}$$

Strain Energy,

$$du = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$du = \frac{1}{2} \times \tau \times \gamma \times \text{Volume}$$

$$du = \frac{1}{2} \times \tau \times \left(\frac{\tau}{G}\right) \times dl \times t \times ds$$

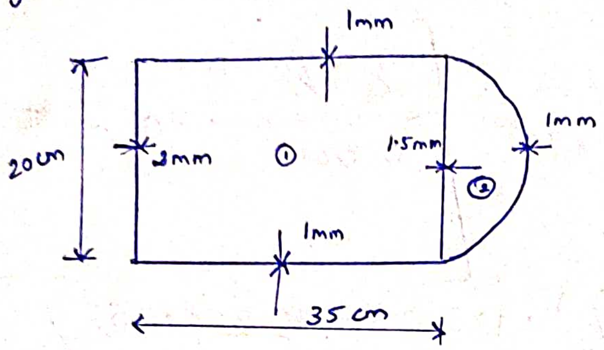
$$du = \frac{1}{2} \times \frac{\tau^2}{G} \times dl \times t \times ds$$

$$du = \frac{\tau^2 t}{2G} \int dl \times ds$$

Angle of twist per unit length $= \dfrac{\text{Strain Energy}}{\text{Torque}}$

$$\boxed{\text{i.e. } \theta = \frac{1}{2AG} \frac{q}{t} \int ds}$$

1. Find the shear flow per unit length of a two cell tube both made of Al, and $G = 2.69 \times 10^{10}$ Pa subjected to a torque of 90000 Ncm.



Given

$G = 2.69 \times 10^{10}$ Pa

$T = 90000$ Ncm

$= 900$ Nm

$T = 2A_1 q_1 + 2A_2 q_2$

$$900 = 2 \times [0.2 \times 0.35] q_1 + 2 \left[\frac{\pi \times (0.1)^2}{2}\right] q_2$$

$$0.14 q_1 + 0.0314 q_2 = 900 \quad \text{——} (1)$$

Angle of twist is

$$\theta = \frac{1}{2AG} \frac{q}{t} \int ds$$

$$\theta_1 = \frac{1}{2 \times 0.2 \times 0.35 \times 2.69 \times 10^{10}} \left[\frac{q_1}{0.002} \times 0.2 + \frac{q_1}{0.001} \times 0.35 + \frac{q_1}{0.001} \times 0.35 + \frac{q_1 - q_2}{0.0015} \times 0.2\right]$$

$$\theta_1 = 2.655 \times 10^{-10} \left[100 q_1 + 700 q_1 + 133.33 (q_1 - q_2)\right]$$

$$\theta_1 = 2.655 \times 10^{-10} \left[800q_1 + 133.33q_1 - 133.33q_2\right]$$

$$\theta_1 = 2.477 \times 10^{-7} q_1 - 3.539 \times 10^{-8} q_2$$

$$\theta_1 = 2.477 \times 10^{-7} q_1 - 0.3539 \times 10^{-7} q_2 \quad \text{—— (2)}$$

$$\theta_2 = \frac{1}{2 \times \frac{\pi \times (0.1)^2}{2} \times 2.69 \times 10^{10}} \left[\frac{q_2 - q_1}{0.0015} \times 0.2 + \frac{q_2}{0.001} \times \pi \times 0.1\right]$$

$$\theta_2 = 1.183 \times 10^{-9} \left[(q_2 - q_1) \times 133.33 + \cancel{3}314.159 q_2\right]$$

$$\theta_2 = 1.183 \times 10^{-9} \left[133.33 q_2 - 133.33 q_1 + 314.159 q_2\right]$$

$$\theta_2 = 1.183 \times 10^{-9} \left[447.489 q_2 - 133.33 q_1\right]$$

$$\theta_2 = -1.577 \times 10^{-7} q_1 + 5.293 \times 10^{-7} q_2 \quad \text{—— (3)}$$

we know,

$$\theta = \theta_1 = \theta_2$$

$$\therefore (2.477 q_1 - 0.3539 q_2) \times \cancel{10^{-7}} = (-1.577 q_1 + 5.293 q_2) \cancel{10^{-7}}$$

$\cancel{2.4}$ ~~2.477q₁ − 0.3539~~

$$2.477 q_1 + 1.577 q_1 - 0.3539 q_2 - 5.293 q_2 = 0$$

$$4.054 q_1 - 5.646 q_2 = 0 \quad \text{—— (4)}$$

Solving eqns (1) & (4)

$$0.14 q_1 + 0.0314 q_2 = 900 \quad \text{—— (1)}$$

$$4.054 q_1 - 5.646 q_2 = 0 \quad \text{—— (4)}$$

$q_1 = 5536.88\ N/m$

$q_2 = 3975.65\ N/m$

$\because \theta_1 = \theta_2 = \theta$

Sub. q_1 & q_2 in (2)

we get.

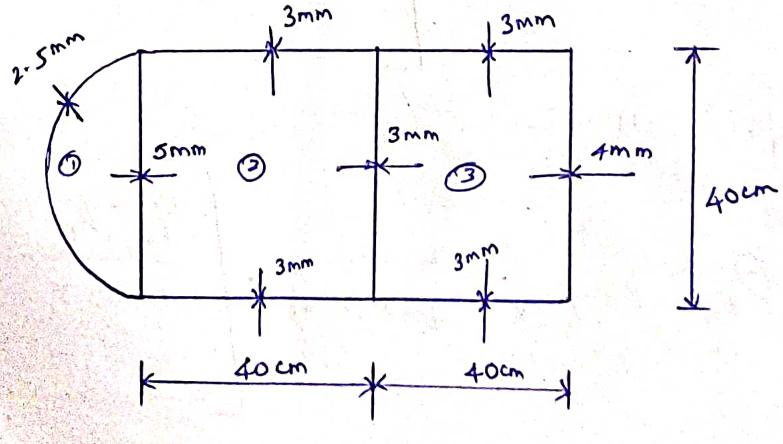
$\theta_1 = 2.477 \times 10^{-7} \times 5536.88 - 0.3539 \times 10^{-7} \times 3975.65$

$\theta_1 = 1.2307 \times 10^{-3}$ rad

$\theta = \theta_1 = 0.07051^\circ$

$1\ Rad \times \frac{180}{\pi} \Rightarrow deg.$

2. Compute the Shear flow distribution for a 3 Cell Section which is subjected to a torque load of 19 kNm in anti clock wise direction.



$T = 19\,kNm$ (anticlockwise)

$\therefore T = -19 \times 10^3\ Nm$

$T = 2A_1q_1 + 2A_2q_2 + 2A_3q_3$

$$-19000 = \left[2\times\pi\frac{(0.2)^2}{2}\times q_1\right] + \left[2\times0.4\times0.4\right]q_2 + \left[2\times0.4\times0.4\right]q_3$$

$$\boxed{0.125q_1 + 0.32q_2 + 0.32q_3 = -19000 \quad \text{--- (1)}}$$

we know, angle of twist is $\theta = \frac{1}{2AG}\frac{q}{t}\int ds$

$\therefore$ for cell 1

$$\theta_1 = \frac{1}{2\times\frac{\pi(0.2)^2}{2}G}\left[\frac{q_1}{0.0025}(\pi\times0.2) + \frac{q_1-q_2}{0.005}\times0.4\right]$$

$$\theta_1 = \frac{7.957}{G}\left[251.327q_1 + 80(q_1-q_2)\right]$$

$$\theta_1 = \frac{7.957}{G}\left[331.327q_1 - 80q_2\right] \quad \text{--- (2)}$$

for cell 2,

$$\theta_2 = \frac{1}{2\times0.4\times0.4\times G}\left[\frac{q_2-q_1}{0.005}\times0.4 + \frac{q_2}{0.003}\times0.4 + \frac{q_2-q_3}{0.003}\times0.4 + \frac{q_2}{0.003}\times0.4\right]$$

$$\theta_2 = \frac{3.125}{G}\left[-80q_1 + 80q_2 + 266.67q_2 + 133.33q_2 - 133.33q_3\right]$$

$$\theta_2 = \frac{3.125}{G}\left[-80q_1 + 479.99q_2 - 133.33\,q_3\right] \quad \text{——} \; (3)$$

For cell 3,

$$\theta_3 = \frac{1}{2 \times 0.4 \times 0.4 \times G}\left[\frac{q_3 - q_2}{0.003} \times 0.4 + \frac{q_3}{0.003} \times 0.4 + \frac{q_3}{0.004} \times 0.4 + \frac{q_3}{0.003} \times 0.4\right]$$

$$\theta_3 = \frac{3.125}{G}\left[133.33\left(q_3 - q_2\right) + 266.66\,q_3 + 100q_3\right]$$

$$\theta_3 = \frac{3.125}{G}\left[-133.33\,q_2 + 499.99\,q_3\right] \quad \text{——} \; (4)$$

Angle of twist is constant throughout.

$$\therefore \theta = \theta_1 = \theta_2 = \theta_3$$

Let us consider, $\theta_1 = \theta_2$

$$\therefore \frac{7.957}{\cancel{G}}\left[331.327\,q_1 - 80q_2\right] = \frac{3.125}{\cancel{G}}\left[-80q_1 + 479.99q_2 - 133.33q_3\right]$$

$$\boxed{923.558q_1 - 683.67\,q_2 + 133.33\,q_3 = 0} \quad \text{——} \; (5)$$

also, $\theta_2 = \theta_3$

$$\therefore \frac{3.125}{G}\left[-80q_1 + 479.99q_2 - 133.33q_3\right] = \frac{3.125}{G}\left[-133.33\,q_2 + 499.99q_3\right]$$

$$-80q_1 + 479.99q_2 + 133.33q_3 - 133.33q_3 - 499.99q_3 = 0$$

$$\boxed{\text{ie} \quad -80q_1 + 613.32q_2 - 633.32q_3 = 0} \quad — (6)$$

By solving (1), (5) & (6) we get q_1, q_2, q_3

$$0.1256q_1 + 0.32q_2 + 0.32q_3 = -19000 \quad — (1)$$

$$923.558q_1 - 683.67q_2 + 133.33q_3 = 0 \quad — (5)$$

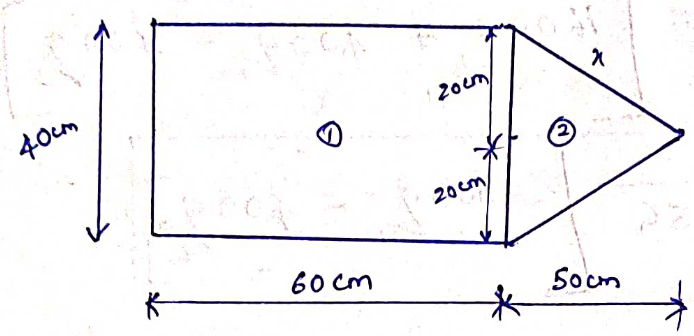
$$-80q_1 + 613.32q_2 - 633.32q_3 = 0 \quad — (6)$$

$q_1 = -17039.059$ N/m

$q_2 = -27859.656$ N/m

$q_3 = -24827.512$ N/m

3. Find the Shear flow per unit length of a two cell tube both made of Al and subjected to a torque of 60 kNm $G = 2.69 \times 10^{10}$ Pa,



To find x we use Pythagoras theorem

$x^2 = 50^2 + 20^2$

$x = \sqrt{2900}$

$x = 53.85$ cm

$= 0.5385$ m

Given,

$T = 60$ kNm

$= 60000$ Nm

Since the wall thickness is not given, let us assume it to be unit thickness $\Rightarrow t = 1$ mm

$G = 2.69 \times 10^{10}$ Pa

$T = 2A_1q_1 + 2A_2q_2$

$60000 = (2 \times 0.4 \times 0.6)q_1 + \left(2 \times \frac{1}{2} \times 0.4 \times 0.5\right) q_2$

$\boxed{\Rightarrow 0.48q_1 + 0.20q_2 = 60000}$ — ①

we know angle of twist,

$$\theta = \frac{1}{2AG} \frac{q}{t} \int ds$$

for cell ①

$$\theta_1 = \frac{1}{2\times0.4\times0.6\,G}\left[\frac{q_1}{0.001}\times0.4 + \frac{q_1}{0.001}\times0.6 + \frac{q_1-q_2}{0.001}\times0.4 + \frac{q_1}{0.001}\times0.6\right]$$

$$\theta_1 = \frac{1}{0.48G}\left[1600q_1 + 400q_1 - 400q_2\right]$$

$$\boxed{\theta_1 = \frac{1}{0.48G}\left[2000q_1 - 400q_2\right]} \quad \text{—— ②}$$

Consider cell ②

$$\theta_2 = \frac{1}{2\times\left(\frac{1}{2}\times0.4\times0.5\right)\times G}\left[\frac{q_2-q_1}{0.001}\times0.4 + \frac{q_2}{0.001}\times0.5385 + \frac{q_2}{0.001}\times0.5385\right]$$

$$\theta_2 = \frac{1}{0.2G}\left[400q_2 - 400q_1 + 1077q_2\right]$$

$$\boxed{\theta_2 = \frac{1}{0.2G}\left[-400q_1 + 1477q_2\right]} \quad \text{—— ③}$$

We know angle of twist is constant throughout.

$$\therefore\ \theta_1 = \theta_2$$

$$\frac{1}{0.48G}\left[2000q_1 - 400q_2\right] = \frac{1}{0.2G}\left[-400q_1 + 1477q_2\right]$$

$$4166q_1 - 833.2q_2 = -2000q_1 + 7385q_2 = 0$$

$$\boxed{6166q_1 - 8218.2q_2 = 0} \quad \text{--- (4)}$$

By solving (1) & (4) we get q_1 & q_2

$$0.48q_1 + 0.20q_2 = 60000 \quad \text{--- (1)}$$

$$6166q_1 - 8218.2q_2 = 0 \quad \text{--- (4)}$$

$$q_1 = 95229.45 \text{ N/m}$$

$$q_2 = 71,449.31 \text{ N/m}$$

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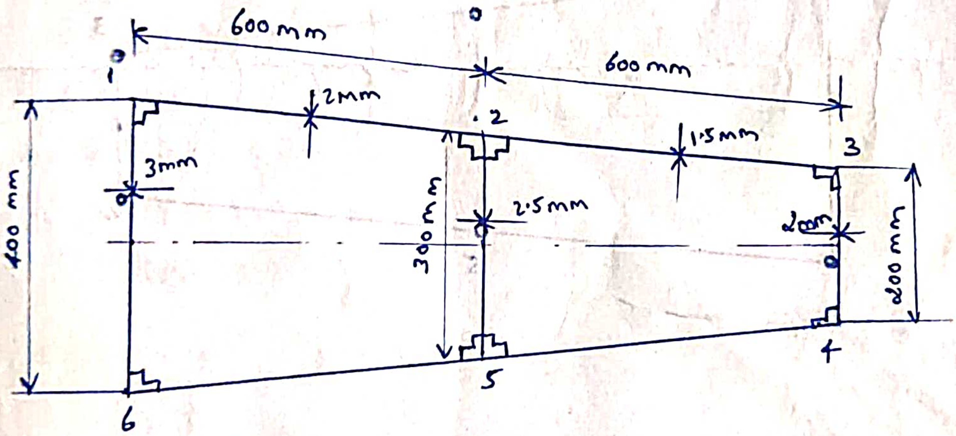
Structural Idealization.

1. Part of a wing section is in the form of the two cell box shown in which the vertical spars are connected to the wing skin through angle sections all having a cross sectional area of 300 mm². Idealize the section into an arrangement of direct stress carrying booms and shear stress only carrying panels suitable for resisting bending moments in a vertical plane. Position the booms at the spar / skin junctions.



$$B_1 = \frac{tb}{6}\left(2 + \frac{\sigma_6}{\sigma_1}\right) + \frac{tb}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right) + 300$$

$$= \frac{3 \times 400}{6}\left(2 + \frac{-200}{+200}\right) + \frac{2 \times 600}{6}\left(2 + \frac{150}{200}\right) + 300$$

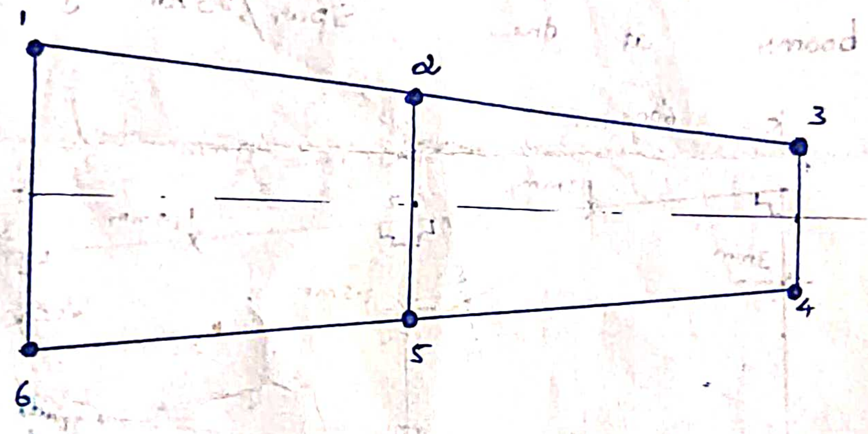
$$B_6 = B_1 = 1050\,mm^2$$

$$B_1 = 300 + 300 + \frac{tb}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right) + \frac{tb}{6}\left(2 + \frac{\sigma_3}{\sigma_2}\right) + \frac{tb}{6}\left(2 + \frac{\sigma_5}{\sigma_2}\right)$$

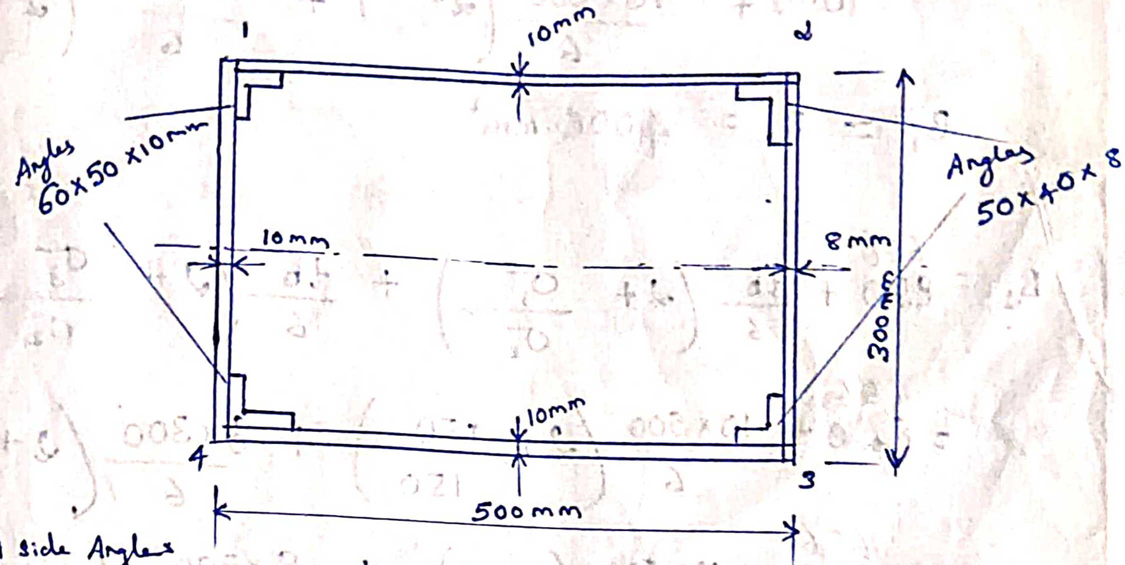
$$B_2 = B_5 = 1791 \text{ mm}^2$$

$$B_4 = B_3 = 300 + \frac{tb}{6}\left(2 + \frac{\sigma_2}{\sigma_3}\right) + \frac{tb}{6}\left(2 + \frac{\sigma_4}{\sigma_3}\right)$$

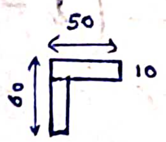
$$= 891 \text{ mm}^2$$



2. Idealize the box section shown in Fig. ~~[illegible]~~ into an arrangement of direct stress carrying booms positioned at the four corners and panels which are assumed to carry only shear stress.

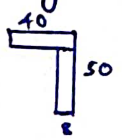


Left side Angles



$$\text{Area} = (50\times10) + (50\times10)$$
$$= 500 + 500$$
$$= 1000\ \text{mm}^2$$

Right side Angles.



$$\text{Area} = \cancel{(40\times10)+(42\times10)} \quad (40\times8)+(42\times8)$$
$$= \cancel{400+420} \quad = 320+336$$
$$\text{Area} = \cancel{820\ \text{mm}^2} \quad = 656\ \text{mm}^2$$

$$B_1 = 1000 + \frac{tb}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right) + \frac{tb}{6}\left(2 + \frac{\sigma_4}{\sigma_1}\right)$$

$$= 1000 + \frac{10 \times 500}{6}\left(2 + \frac{150}{150}\right) + \frac{10 \times 300}{6}\left(2 + \frac{-150}{150}\right)$$

$$= 1000 + \frac{10 \times 500}{6}(2 + 1) + \frac{10 \times 300}{6}(2 - 1)$$

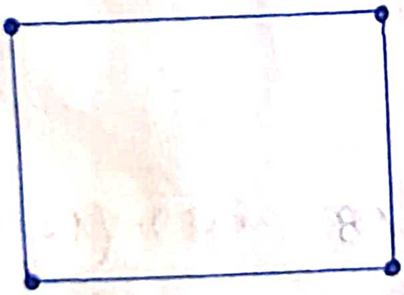
$$B_1 = B_4 = 4000 \text{ mm}^2$$

$$B_2 = \overset{656}{\cancel{820}} + \frac{tb}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right) + \frac{tb}{6}\left(2 + \frac{\sigma_3}{\sigma_2}\right)$$

$$= \overset{656}{\cancel{820}} + \frac{10 \times 500}{6}\left(2 + \frac{150}{150}\right) + \frac{8 \times 300}{6}\left(2 + \frac{-150}{150}\right)$$

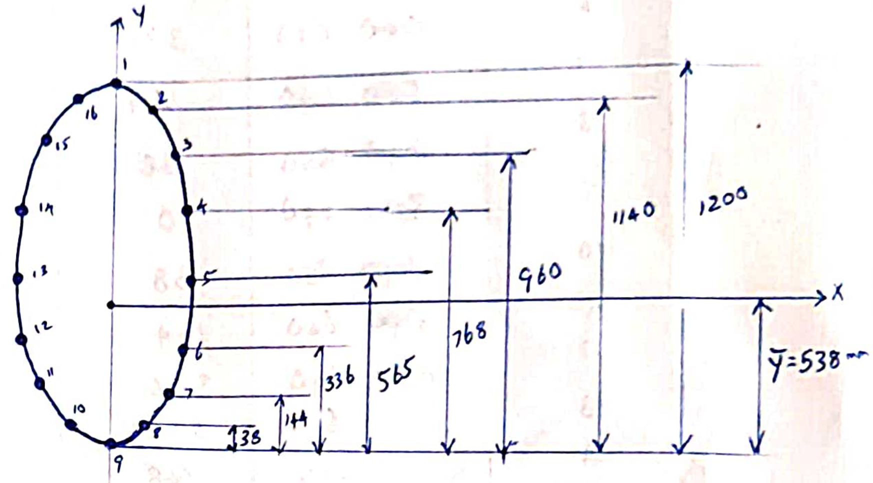
$$= \overset{656}{\cancel{820}} + \frac{10 \times 500}{6}(2 + 1) + \frac{8 \times 300}{6}(2 - 1)$$

$$B_2 = B_3 = 3556 \text{ mm}^2$$



Direct Stress in Idealized Structure

1. A fuselage section as shown in figure is subjected to a bending moment of 100 kNm applied in the vertical plane of symmetry. If the section has been completely idealized into a combination of direct stress carrying booms and shear stress only carrying panels, determine the direct stress in each boom.



$M_x = 100\,kNm$

$M_y = 0$

The section is symmetrical to y axis,

$\therefore I_{xy} = 0$

$$\sigma = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right)y$$

$$= \frac{M_x I_{yy}}{I_{xx} I_{yy}} \times y$$

$$\sigma = \frac{M_x}{I_{xx}}\, y$$

$$\bar{y} = \frac{\Sigma ay}{\Sigma a}$$

Booms	Area mm^2	y mm	h for I_{xx} $y-\bar{y}$ mm	I_{xx} mm^4
1	~~640~~ 640	1200	662	280×10^6
2	~~600~~ 600	1140	602	217×10^6
3	~~600~~ 600	960	422	106×10^6
4	~~620~~ 600	768	230	31×10^6
5	~~640~~ 620	565	27	0.4×10^6
6	~~640~~ 640	336	-202	26×10^6
7	~~850~~ 640	144	-394	99×10^6
8	~~640~~ 850	38	-500	212×10^6
9	~~850~~ 640	0	-538	185×10^6
10	~~640~~ 850	38	-500	212×10^6
11	~~640~~ 640	144	-394	212×10^6
12	~~620~~ 640	336	-202	99×10^6
13	620	565	27	26×10^6
14	600	768	230	34×10^6
15	600	960	422	31×10^6
16	600	1140	602	106×10^6
				217×10^6

$$\bar{y} = \frac{(640 \times 1200) + (600 \times 1140) + (600 \times 960) + (600 \times 768) + (620 \times 565) + (640 \times 336) + (640 \times 144) + (850 \times 38) + (640 \times 0) + (850 \times 38) + (640 \times 144) + (640 \times 336) + (620 \times 565) + (600 \times 768) + (600 \times 960) + (600 \times 1140)}{640 + 600 + 600 + 600 + 620 + 640 + 640 + 850 + 640 + 850 + 640 + 640 + 620 + 600 + 600 + 600}$$

$$\bar{y} = \frac{5589200}{10380} = 538 \text{ mm}$$

$$I_{xx_1} = \frac{bd^3}{12} + A_1 h_1^2$$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3} + \dots I_{xx_{16}}$$

$$\Sigma I_{xx} = I_{xx} = 1847.8 \times 10^6 \text{ mm}^4$$

$$\sigma_1 = \frac{M_x}{I_{xx}} \times y_1$$

$$= \frac{100 \times 10^6}{1847 \times 10^6} \times 662$$

$$= 0.0541 \times 662$$

$$\sigma_1 = 35.8 \text{ N/mm}^2$$

$$\sigma_2 = \frac{100 \times 10^6}{1847 \times 10^6} \times 602$$

$$= 32.5 \text{ N/mm}^2$$

$$\sigma_3 = \frac{100 \times 10^6}{1847 \times 10^6} \times 422$$

$$= 22.8 \text{ N/mm}^2$$

$$\sigma_4 = \frac{100 \times 10^6}{1847 \times 10^6} \times 230$$

$$= 12.4 \text{ N/mm}^2$$

$$\sigma_5 = \frac{100 \times 10^6}{1847 \times 10^6} \times 27$$

$$= 1.46 \text{ N/mm}^2$$

$$\sigma_6 = \frac{100 \times 10^6}{1847 \times 10^6} \times -202$$

$$= -10.9 \ N/mm^2$$

$$\sigma_7 = \frac{100 \times 10^6}{1847 \times 10^6} \times -394$$

$$= -21.3 \ N/mm^2$$

$$\sigma_8 = \frac{100 \times 10^6}{1847 \times 10^6} \times -500$$

$$= -27.05 \ N/mm^2$$

$$\sigma_9 = \frac{100 \times 10^6}{1847 \times 10^6} \times -538$$

$$= -29.1 \ N/mm^2$$

By Symmetry (cross section is Symmetrical to y axis)

$\therefore$

$$\sigma_2 = \sigma_{16} = 32.5 \ N/mm^2.$$

$$\sigma_3 = \sigma_{15} = 22.8 \ N/mm^2$$

$$\sigma_4 = \sigma_{14} = 12.4 \ N/mm^2$$

$$\sigma_5 = \sigma_{13} = 1.46 \ N/mm^2$$

$$\sigma_6 = \sigma_{12} = -10.9 \ N/mm^2$$

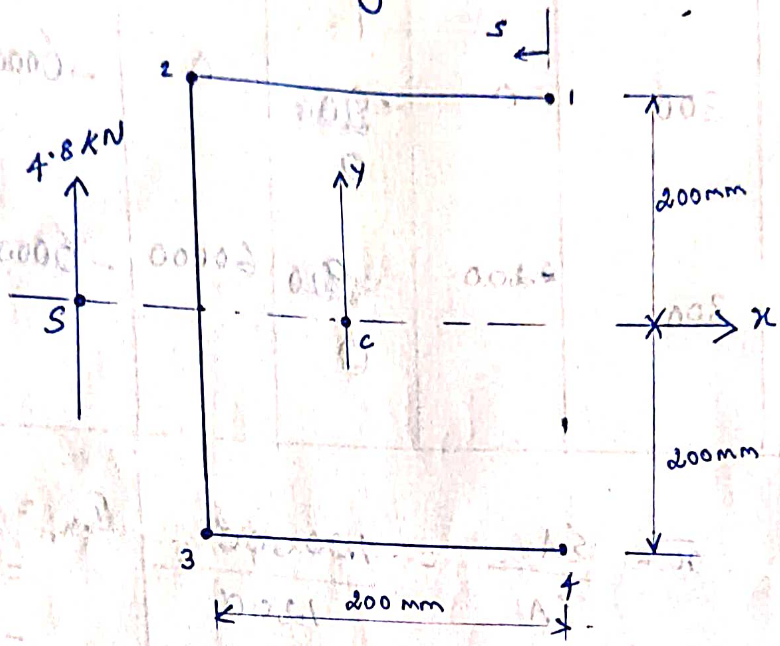
$$\sigma_7 = \sigma_{11} = -21.3 \ N/mm^2$$

$$\sigma_8 = \sigma_{10} = -27.05 \ N/mm^2$$

Calcul

1

Q. Calculate the shear flow distribution in the Channel Section shown in fig. produced by a vertical Shear load of 4·8 kN acting through its shear Centre. Assume that the walls of the section are only effective in resisting Shear Stresses, while the booms, each of area $300\ mm^2$ carry all direct Stresses.



$$q = \left[\frac{S_y \cancel{I_{xy}}^{0} - S_x \cancel{I_{xx}}^{0}}{I_{xx} I_{yy} - I_{xy}^2}\right] A_i x_i + \left[\frac{S_x \cancel{I_{xy}} - S_y I_{yy}}{I_{xx} I_{yy} - I_{xy}^2}\right] A_i y_i$$

$S_y = 4 \cdot 8\ kN$

$= 4 \cdot 8 \times 1000$

$S_y = 4800\ N$

$S_x = 0$

$\because$ The Section is symmetrical to one axis, $I_{xy} = 0$

$$\therefore\ q = \left[\frac{-S_y \cancel{I_{yy}}}{I_{xx} \cancel{I_{yy}}}\right] A_i y_i$$

$$q = \frac{-S_y}{I_{xx}} A_i y_i$$

Booms	Area mm^2	x	y	Ax	Ay	h for I_{xx} $y - \bar{y}$
1	300	200	~~4~~00	60000	~~6~~0000	200
2	300	0	~~4~~00	0	~~6~~0000	200
3	300	0	~~200~~ 0	0	-60000	$-~~4~~00$ 2
4	300	200	~~200~~ 0	60000	-60000	$-~~4~~00$ 2

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{12000\cancel{0}}{120\cancel{0}}$$

$= 100\,mm$

$$\cancel{\bar{y} = \frac{24000\cancel{0}}{120\cancel{0}}}$$

$\cancel{\bar{y} = 200}$

$$\bar{y} = \frac{24000}{1200}$$

$\bar{y} = 200$

$I_{xx_1} = Ah^2$

$= 300 \times 200^2$

$= 12\,000\,000\,mm^4$

$I_{xx_2} = 300 \times 200^2$

$= 12\,000\,000\,mm^4$

$I_{xx_3} = 300 \times -200^2$

$= 12\,000\,000\,mm^4$

$I_{xx_4} = 300 \times (-200)^2$

$= 12\,000\,000\,mm^4$

$I_{xx} = 48000000$

$$q_{12} = \frac{-48\cancel{00}}{48\cancel{000000}} \times 3\cancel{00} \times 2\cancel{00}$$

$= -6\,N/mm$

$$q_{23} = \frac{-4800}{4800000} \times 300 \times 200 = -6 + q_{12}$$

$= -12\,N/mm$

$$q_{34} = q_{23} + \frac{-4800}{48000000} \times 300 \times -200$$

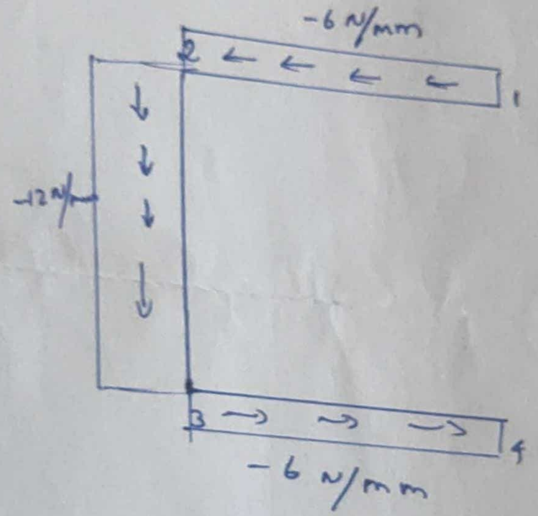
$$= -12 + 6$$

$$q_{34} = -6 \text{ N/mm}$$

$$q_{41} = -6 + \left[-\frac{4800}{48000000} \times 300 \times -200 \right]$$

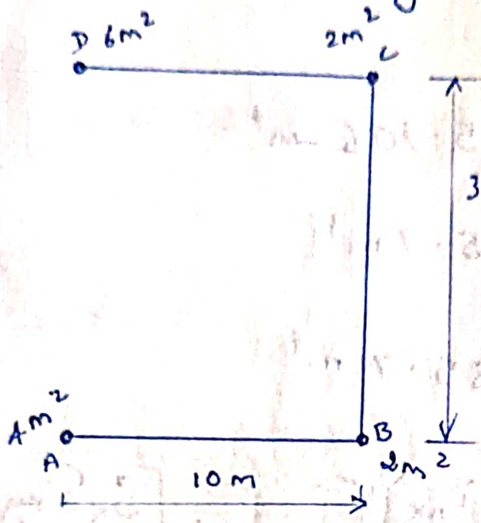
$$= -6 + 6$$

$$= 0$$



Shear flow distribution in idealized structures.

1. Calculate the shear flow distribution in the channel section shown in fig. produced by a vertical shear load of 1 kN. Assume that the walls of the section are only effective in resisting shear stress while the booms carry all direct stress.



Booms	Area m^2	x	y	Ax	Ay
A	4	0	0	0	0
B	2	10	0	20	0
D	6	0	30	0	180
C	2	10	30	20	60

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{40}{14} = 2.857\,m$$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{240}{14} = 17.142\,m$$

Booms	Area	h for I_{xx} $y - \bar{y}$	h for I_{yy} $x - \bar{x}$	Ah^2 I_{xx}	I_{yy}	
A	4	-17.142	-2.857	1175	32	
B	2	-17.142	7.143	587	102	-244
C	2	12.858	7.143	330	102	183
D	6	12.858	-2.857	992	48	-220

$$I_{xx} = \Sigma A h^2 = 3085.906 \ m^4$$

$$I_{yy} = \Sigma A h^2 = 285.7 \ m^4$$

$$I_{xy} = \Sigma A xy = -85.71 \ m^4$$

$$q = \left[\frac{S_y I_{xy} - S_x I_{xx}}{I_{xx} I_{yy} - I_{xy}^2}\right] A_i x_i + \left[\frac{S_x I_{xy} - S_y I_{yy}}{I_{xx} I_{yy} - I_{xy}^2}\right] A_i y_i$$

$$S_x = 0, \quad S_y = 1 \ kN = 1000 \ N$$

$$q = \left[\frac{(1000 \times -85.71) - 0}{(3085 \times 285) - 85.7}\right] A_i x_i + \left[\frac{0 - (1000 \times 285.7)}{(3085 \times 285) - 85.71}\right] A_i y_i$$

$$q = -0.098 A_i x_i - 0.326 A_i y_i$$

$$q_{AB} = (-0.098 \times 4 \times -2.857) + (-0.326 \times 4 \times -17.142)$$

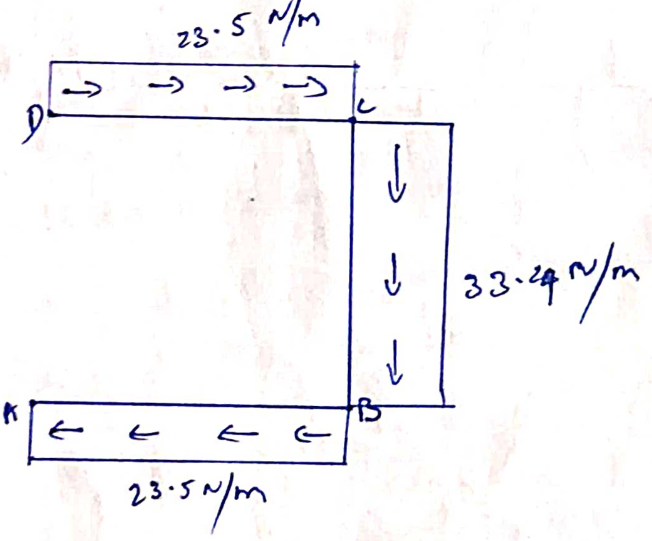
$$= 23.473 \ N/m$$

$$q_{BC} = (-0.098 \times 2 \times 7.143) + (-0.326 \times 2 \times -17.142) + q_{AB}$$

$$= 33.29 \ N/m$$

$$q_{CD} = (-0.098 \times 2 \times 7.143) + (-0.326 \times 2 \times 12.858) + q_{BC}$$

$$= 23.506 \ N/m$$



To find Shear Centre

$$S_y \times e_x = 23.5 \times 10 \times 30 + 33.29 \times 30 \times 10$$

$$e_x = 17.03m$$

Shear flow in Closed Section beam.

$$q = \left[\frac{S_y I_{xy} - S_x I_{xx}}{I_{xx} I_{yy} - I_{xy}^2}\right] A_i x_i + \left[\frac{S_x I_{xy} - S_y I_{yy}}{I_{xx} I_{yy} - I_{xy}^2}\right] A_i y_i + q_{s,0}$$

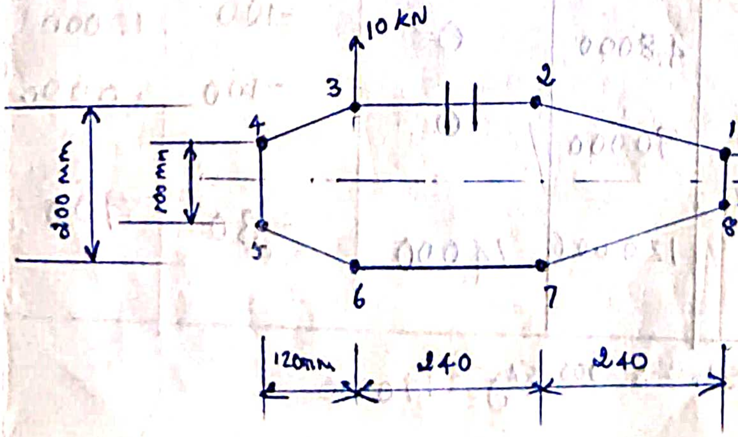
The moment of the Section will be balanced at the open Section when it is closed.

$$T = 2 A q_{s,0}$$

1. A thin walled single cell beam shown in Fig. has been idealized into a combination of direct stress carrying booms and shear stress only carrying shear walls. If the section supports a vertical shear load of 10 kN acting in a vertical plane through booms 3 & 6, Calculate the shear flow distribution around the section.

Boom areas: $B_1 = B_8 = 200\,mm^2$, $B_2 = B_7 = 250\,mm^2$,

$B_3 = B_6 = 400\,mm^2$, $B_4 = B_5 = 100\,mm^2$



$$q = \left[\frac{S_y \cancel{I_{xy}}^{0} - S_x \cancel{I_{xx}}}{I_{xx} I_{yy} - I_{xy}^2}\right] A_i x_i + \left[\frac{S_x \cancel{I_{xy}}^{0} - S_y I_{yy}}{I_{xx} I_{yy} - \cancel{I_{xy}^2}^{0}}\right] A_i y_i + q_{s,0}$$

$\because$ the Section is Symmetrical to one axis $I_{xy} = 0$

$S_y = 10 \times 10^3\ N$

$\therefore S_x = 0$

$$\therefore q = \frac{-S_y \cancel{I_{yy}}}{I_{xx} \cancel{I_{yy}}} A_i y_i + q_{s,0}$$

$$q = -\frac{S_y}{I_{xx}} A_i y_i + q_{s,0}$$

ie $q = q_b + q_{s,0}$

Boom	Area	x	y	Ax	Ay	h for I_{xx} $y - \bar{y}$	h^2	Ah^2
1	200	600	130	120000	26000	30	900	180000
2	250	360	200	90,000	50000	100	10000	2500000
3	400	120	200	48000	80000	100	10000	4000000
4	100	0	150	0	15000	50	2500	250000
5	100	0	50	0	5000	-50	2500	250000
6	400	120	0	48000	0	-100	10000	4000000
7	250	360	0	90000	0	-100	10000	2500000
8	200	600	70	120000	14000	-30	900	180000

$\Sigma A = 1900$ $\qquad \Sigma Ax = 516000 \quad \Sigma Ay = 190000$

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{516000}{1900} = 271\ mm$$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{19000}{1900} = 100\ mm$$

$I_{xx} = \Sigma A h^2$

$= 13860000$

$I_{xx} = 13.86 \times 10^6 \ mm^4$

$q_{b23} = 0$

$q_{b34} = \dfrac{-10 \times 10^3}{13.86 \times 10^6} \times 400 \times 100$

$= \dfrac{-400}{13.86}$

$q_{34} = -28.9 \ N/mm$

$q_{45} = \dfrac{-10 \times 10^3}{13.86 \times 10^6} \times 100 \times 50 + q_{34}$

$= -3.60 + (-28.9)$

$q_{45} = -32.5 \ N/mm$

$q_{56} = q_{34} = -28.9 \ N/mm$

$q_{67} = q_{23} = 0$

$q_{21} = \dfrac{-10 \times 10^3}{13.86 \times 10^6} \times 250 \times 100$

$q_{21} = -18.03 \ N/mm$

$q_{18} = \dfrac{-10 \times 10^3}{13.86 \times 10^6} \times 200 \times 30$

$= -22.4 \ N/mm$

$q_{87} = q_{1} = -18.03 N/mm$

The moment at any point of the Section will be balanced when it is closed.

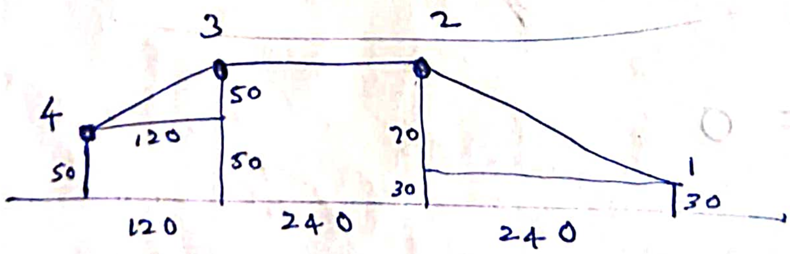
$$T = 2A\,q_{s,0}$$

take moment about the intersection of Shear load and line of Symmetry,

$$q_{18} \times 60 \times 480 + q_{21} \times 240 \times 100 + q_{87} \times 240 \times 30 + q_{23} \times 240 \times 100 + q_{76} \times 240 \times 100 - q_{34} \times 120 \times 100 - q_{56} \times 120 \times 50 - q_{45} \times 100 \times 120 = T$$

$$\therefore T = 2A\,q_{s,0}$$

to find Area,



$$A = \left[(120 \times 50) + \left(\tfrac{1}{2} \times 120 \times 50\right) + (240 \times 100) + \left(\tfrac{1}{2} \times 240 \times 70\right) + (30 \times 240)\right] \times 2$$

$$\therefore A = 97200 \text{ mm}^2$$

$$\therefore T = 2 \times 97200\, q_{s,0}$$

Take moment with respect to any point on the beam to find T and substitute in the above equation to find qs0
Add qso with all the basic shear flow values.

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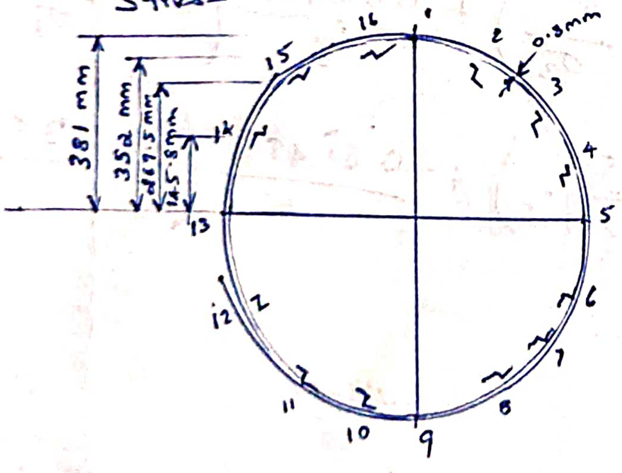
Unit IV

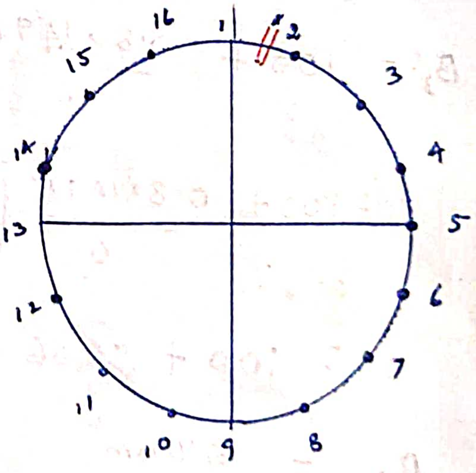
Stress Analysis of Aircraft Component.

Fuselages:

a. Bending.

1. The fuselage of a lighter passenger carrying aircraft has the circular section shown in Fig. ~~[illegible]~~ The Cross-Sectional area of each Stringer is 100 mm^2 and the vertical distance given in fig. are to the mid line of the Section wall at the corresponding Stringers Position. If the fuselage is subjected to a bending moment of 200 kNm applied in the vertical plane of Symmetry, at this Section, Calculate the direct Stress distribution.





$$B = \frac{tb}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right)$$

$t = 0.8\,mm$

$$b = \frac{2\pi r}{16} = \frac{2 \times 3.14 \times 381}{16} = 149.6\,mm$$

$$B_1 = 100 + \frac{0.8\times 149.6}{6}\left(2+\frac{\sigma_2}{\sigma_1}\right) + \frac{0.8\times 149.6}{6}\left(2+\frac{\sigma_{12}}{\sigma_1}\right)$$

$$= 100 + \frac{0.8\times 149.6}{6}\left(2+\frac{352}{381}\right) + \frac{0.8\times 149.6}{6}\left(2+\frac{352}{381}\right)$$

$$= 100 + 58.244 + 58.244$$

$$B_1 = 216.48\ mm^2$$

$$B_2 = 100 + \frac{0.8\times 149.6}{6}\left(2+\frac{\sigma_3}{\sigma_2}\right) + \frac{0.8\times 149.6}{6}\left(2+\frac{\sigma_1}{\sigma_2}\right)$$

$$= 100 + \frac{0.8\times 149.6}{6}\left(2+\frac{269.5}{352}\right) + \frac{0.8\times 149.6}{6}\left(2+\frac{381}{352}\right)$$

$$= 100 + 55.26 + 61.46$$

$$B_2 = 216\ mm^2$$

$$B_3 = 100 + \frac{0.8\times 149.6}{6}\left(2+\frac{\sigma_4}{\sigma_3}\right) + \frac{0.8\times 149.6}{6}\left(2+\frac{\sigma_2}{\sigma_3}\right)$$

$$= 100 + \frac{0.8\times 149.6}{6}\left(2+\frac{145.8}{269.5}\right) + \frac{0.8\times 149.6}{6}\left(2+\frac{352}{269.5}\right)$$

$$= 100 + 50.66 + 65.92$$

$$B_3 = 216\ mm^2$$

$$B_4 = 100 + \frac{0.8\times 149.6}{6}\left(2+\frac{\sigma_5}{\sigma_4}\right) + \frac{0.8\times 149.6}{6}\left(2+\frac{\sigma_3}{\sigma_4}\right)$$

$$= 100 + \frac{0.8\times 149.6}{6}\left(2+\frac{0}{145.8}\right) + \frac{0.8\times 149.6}{6}\left(2+\frac{269.5}{145.8}\right)$$

$$= 100 + 39.88 + 76.73$$

$$B_4 = 216\ mm^2$$

$$B_5 = 100 + \frac{0.8 \times 149.6}{6}\left(2 + \frac{\sigma_6}{\sigma_5}\right) + \frac{0.8 \times 149.6}{6}\left(2 + \frac{\sigma_4}{\sigma_5}\right)$$

$= 100\,mm^2$ + undefined.

$B_1 = B_9 = 216\ mm^2$

$B_2 = B_{16} = B_8 = B_{10} = 216\,mm^2$

$B_3 = B_{15} = B_7 = B_{11} = 216\,mm^2$

$B_4 = B_{14} = B_6 = B_{12} = 216\,mm^2$

$B_5 = B_{13} = 100\,mm^2$ + undefined.

Direct stress,

$$\sigma_z = \left[\frac{\cancel{M_y}^{\,0} I_{xx} - M_x \cancel{I_{xy}}^{\,0}}{I_{xx} I_{yy} - I_{xy}^2}\right] x + \left[\frac{M_x I_{yy} - \cancel{M_y}^{\,0} I_{xy}}{I_{xx} I_{yy} - \cancel{I_{xy}^2}^{\,0}}\right] y$$

$I_{xy} = 0$

$M_y = 0$

$$\therefore \quad \sigma = \frac{M_x \cancel{I_{yy}}}{I_{xx} \cancel{I_{yy}}} \times y$$

$$\sigma = \frac{M_x}{I_{xx}}\, y$$

$I_{xx} = [2 \times 216.6 \times 381.0^2] + [4 \times 216.6 \times 352.0^2] + [4 \times 216.6 \times 269.5^2]$
$+ [4 \times 216.7 \times 145.8^2] = 2.52 \times 10^8\ mm^4$

(7) 6.

Booms	Area	y (h)	I_{xx} (10^6) Ah^2	σ_z
1	216	381	31	314
2	216	352	26	290
3	216	269.5	15	222
4	216	145.8	4	120.49
5	216	0	0	0
6	216	-145.8	4	-120.49
7	216	-269.5	15	-222
8	216	-352	26	-290
9	216	-381	31	-314
10	216	-352	26	-290
11	216	-269.5	15	-222
12	216	-145.8	4	-120.49
13	216	0	0	0
14	216	145.8	4	120.49
15	216	269.5	15	222
16	216	352	26	290

$$I_{xx} = \Sigma A h^2$$
$$= 242 \times 10^6$$
$$= 2.42 \times 10^8 \text{ mm}^4$$

$$M_x = 200 \text{ kN-m}$$
$$= 200 \times 10^3 \text{ Nm}$$
$$= 200 \times 10^6 \text{ N-mm}$$

$$\sigma_1 = \frac{M_x\, y_1}{I_{nn}}$$

$$= \frac{200 \times 10^6 \times 381}{2.42 \times 10^8}$$

$$= \frac{76200 \times 10^6}{2.42 \times 10^8} = \frac{7.62 \times 10^{10}}{2.42 \times 10^8}$$

$$= 3.14 \times 10^2$$

$$\cancel{\sigma_1 = 0.314} \quad \sigma_1 = 314 \ N/mm^2$$

$$\sigma_2 = \frac{200 \times 10^6 \times 352}{2.42 \times 10^8}$$

$$= \frac{70400 \times 10^6}{2.42 \times 10^8}$$

$$\sigma_2 = 290 \ N/mm^2$$

$$\sigma_3 = \frac{200 \times 10^6 \times 269.5}{2.42 \times 10^8}$$

$$= 222 \ N/mm^2$$

$$\sigma_4 = \frac{200 \times 10^6 \times 145.8}{2.42 \times 10^8}$$

$$\sigma_4 = 120.49 \ N/mm^2$$

$$\sigma_5 = \frac{200 \times 10^6 \times 0}{2.42 \times 10^8}$$

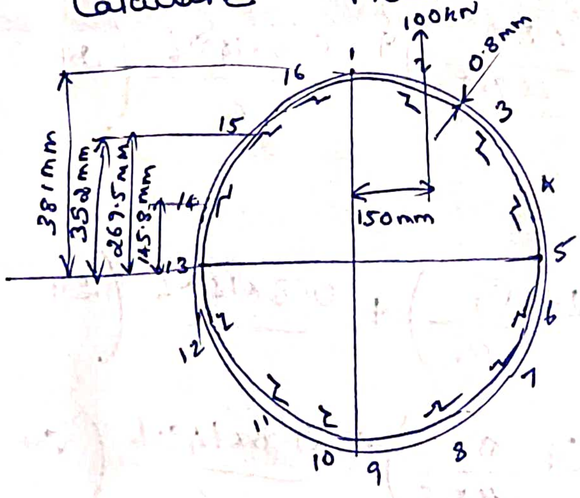
$$\sigma_5 = 0$$

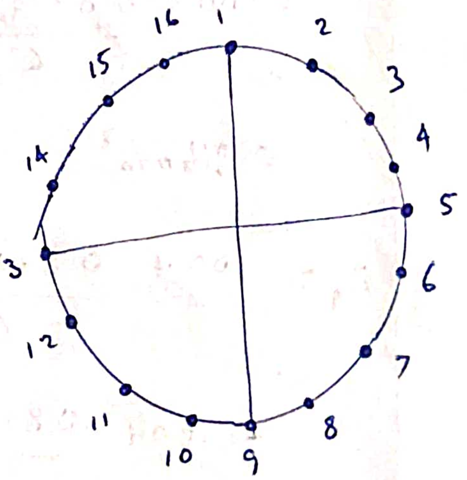
Fuselage.
Shear

1

Problem.

2. The fuselage of a light passenger carrying aircraft has the circular cross section shown in fig. The cross sectional area of each stringer is 100mm² and the vertical distance given in Fig. are to the mid line of the section wall at the corresponding stringer position. If the fuselage is subjected to a vertical shear load of 100kN applied at a distance of 150mm from the vertical axis of symmetry as shown. Calculate the distribution of shear flow.





$$B = \frac{tb}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right)$$

$t = 0.8\,mm$

$$b = \frac{2\pi r}{16} = \frac{2 \times 3.14 \times 381}{16}$$

$= 149.6\,mm$

$B_1 = 100 + \frac{0.8 \times 149.6}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right) + 0.8 \times \frac{149.6}{6}\left(2 + \frac{\sigma_{12}}{\sigma_1}\right)$

$= 100 + \frac{0.8 \times 149.6}{6}\left(2 + \frac{352}{381}\right) + \frac{0.8 \times 149.6}{6}\left(2 + \frac{352}{381}\right)$

$B_1 = 216\ mm^2$

$B_2 = 100 + \frac{0.8 \times 149.6}{6}\left(2 + \frac{\sigma_3}{\sigma_2}\right) + \frac{0.8 \times 149.6}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right)$

$= 100 + \frac{0.8 \times 149.6}{6}\left(2 + \frac{269.5}{352}\right) + \frac{0.8 \times 149.6}{6}\left(2 + \frac{381}{352}\right)$

$= 216\ mm^2$

$B_3 = 100 + \frac{0.8 \times 149.6}{6}\left(2 + \frac{\sigma_4}{\sigma_3}\right) + \frac{0.8 \times 149.6}{6}\left(2 + \frac{\sigma_2}{\sigma_3}\right)$

$= 100 + \frac{0.8 \times 149.6}{6}\left(2 + \frac{145.8}{269.5}\right) + \frac{0.8 \times 149.6}{6}\left(2 + \frac{352}{269.5}\right)$

$= 216\ mm^2$

$B_4 = 100 + \frac{0.8 \times 149.6}{6}\left(2 + \frac{\sigma_5}{\sigma_4}\right) + \frac{0.8 \times 149.6}{6}\left(2 + \frac{\sigma_3}{\sigma_4}\right)$

$= 100 + \frac{0.8 \times 149.6}{6}\left(2 + \frac{0}{145.8}\right) + \frac{0.8 \times 149.6}{6}\left(2 + \frac{269.5}{145.8}\right)$

$B_4 = 216\ mm^2$

B_5 = undefined.

$B_1 = B_9 = 216 mm^2$

$B_2 = B_{16} = B_8 = B_{10} = 216 mm^2$

$B_3 = B_{15} = B_7 = B_{11} = 216 mm^2$

$B_4 = B_{14} = B_6 = B_{12} = 216 mm^2$

$B_5 = B_{13} =$ undefined

basic Shear flow,

$$q_b = \left[\frac{S_y I_{xy} - S_x I_{xx}}{I_{xx} I_{yy} - I_{xy}^2}\right] A_i x_i + \left[\frac{S_x I_{xy} - S_y I_{yy}}{I_{xx} I_{yy} - I_{xy}^2}\right] A_i y_i$$

$S_x = 0, \quad I_{xy} = 0$

$$q_b = 0 + \left[\frac{-S_y \cancel{I_{yy}}}{I_{xx} \cancel{I_{yy}}}\right] A_i y_i$$

$$q_b = -\frac{S_y}{I_{xx}} A_i y_i$$

Total shear flow,

$$q = q_b + q_{s,0}$$

Booms	Area	y	I_{xx} $\times 10^6$
1	216	381	31
2	216	352	26
3	216	269.5	15
4	216	145.8	4
5	undefind	0	0
6	216	-145.8	4
7	216	-269.5	15
8	216	-352	26
9	216	-381	31
10	216	-352	26
11	216	-269.5	15
12	216	-145.8	4
13	undefind	0	0
14	216	145.8	4
15	216	269.5	15
16	216	352	26

$$I_{xx} = \Sigma A h^2$$

$$= 242 \times 10^6$$

$$= 2.42 \times 10^8 \text{ mm}^4$$

Open panned 1-2

$q_{12} = 0$

$$q_{23} = -\frac{S_y}{I_{xx}} A_i y_i + q_{12}$$

$$= \frac{-100 \times 10^3}{2.42 \times 10^8} \times 216 \times 352$$

$$= -31 \text{ N/mm}$$

$$q_{34} = -\frac{S_y}{I_{xx}} A_i y_i + q_{23}$$

$$= -\frac{100 \times 10^3}{2.42 \times 10^8} \times 216 \times 269.5 + q_{23}$$

$$= -24 \cancel{\text{N/mm}} \; -31 \quad = -55 \text{ N/mm}$$

$$q_{45} = -\frac{S_y}{I_{xx}} A_i y_i$$

$$= \frac{-100 \times 10^3}{2.42 \times 10^8} \times 216 \times 145.8 + q_{34}$$

$$= -13 \cancel{\text{N/mm}} + (-55)$$

$$= -68 \text{ N/mm}$$

$\cancel{q_{56}}$

$$q_{56} = 0 - 68 \text{ N/mm}$$

$$= -68 \text{ N/mm}.$$

$$q_{67} = \frac{-100\times10^3}{2.42\times10^8} \times 216 \times -145.8 + q_{56}$$

$$= 12.97 - 68$$

$$= -55 \text{ N/mm}$$

$$q_{78} = \frac{-100\times10^3}{2.42\times10^8} \times 216 \times -269.5 + q_{67}$$

$$= -31 \text{ N/mm}$$

$$q_{89} = \frac{-100\times10^3}{2.42\times10^8} \times 216 \times -352 + q_{78}$$

$$= 31 - 31$$

$$q_{89} = 0$$

$$q_{1-16} = \frac{-100\times10^3}{2.42\times10^8} \times 216 \times 381$$

$$q_{1-16} = -33 \text{ N/mm}$$

$$q_{16-15} = \frac{-100\times10^3}{2.42\times10^8} \times 216 \times 352 - 33$$

$$= -64 \text{ N/mm}$$

$$q_{15-14} = \frac{-100\times10^3}{2.42\times10^8} \times 216 \times 269.5 - 64$$

$$= -23.9 - 64$$

$$= -88 \text{ N/mm}$$

47

$$q_{14-13} = \frac{-100 \times 10^3}{2.42 \times 10^8} \times 145.8 \quad - 88$$

$$= -100 \text{ N/mm}$$

$$q_{13-12} = \frac{-100 \times 10^3}{2.42 \times 10^8} \times \cancel{145.8}\ 0 \quad -100$$

$$= -100 \text{ N/mm}$$

$$q_{12-11} = \frac{-100 \times 10^3}{2.42 \times 10^8} \times -145.8 \quad -100$$

$$= -87 \text{ N/mm}$$

$$q_{11-10} = \frac{-100 \times 10^3}{2.42 \times 10^8} \times -269.5 \quad - 87$$

$$= -63 \text{ N/mm}$$

$$q_{10-9} = \frac{-100 \times 10^3}{2.42 \times 10^8} \times -352 \quad -63$$

$$= -31 \text{ N/mm.}$$

$$T = 2A\,q_{s,0}$$

$$A = \pi \times 381^2$$

$$= 4.55 \times 10^5\ \text{mm}^2$$

Total booms = 16

$$100\times10^3\times150 = \left\{2\times\frac{\text{Area}}{\text{Total no. of booms}} \times \text{Avg height side of } 90\right\} + \left(2\times\text{Area}\times q_{s,0}\right)$$

$$100\times10^3\times150 = 2\times\frac{4.55\times10^5}{16}\times(-240) + \left[2\times4.55\times10^5\times q_{s,0}\right]$$

$$q_{s,0} = 37.8\ \text{N/mm}\quad(\text{or})$$

$$T = 100\times10^3\times150 + q_{23}\times149.6\times352 + q_{34}\times149.6\times269.5$$

$$+ q_{45}\times149.6\times145.8 + q_{56}\times149.6\times 0$$

$$+ q_{67}\times149.6\times145.8 + q_{78}\times149.6\times269.5 + q_{89}\times149.6\times352$$

$$- q_{9\text{-}10}\times149.6\times381 - q_{10\text{-}11}\times149.6\times352 - q_{11\text{-}12}\times149.6\times269.5$$

$$- q_{12\text{-}13}\times149.6\times145.8 - q_{13\text{-}14}\times149.6\times0 - q_{14\text{-}15}\times149.6\times145.8$$

$$- q_{15\text{-}16}\times149.6\times269.6 - q_{16\text{-}1}\times149.6\times352$$

$$T = 100\times10^3\times150 + (-31\times149.6\times352) + (-55\times149.6\times269.5)$$

$$+ (-68\times149.6\times145.8) + (-68\times149.6\times0)$$

$$+ (-55\times149.6\times145.8) + (-31\times149.6\times269.5)$$

$$+ (0) - (-31\times149.6\times381) - (-63\times149.6\times352)$$

$$- (-87\times149.6\times\cancel{352}\ 269.5) - (-100\times149.6\times145.8) - (-100\times149.6\times0)$$

$$- (-88\times149.6\times145.8) - (-64\times149.6\times269.6) - (-33\times149.6\times352$$

$T = 5431737.25$

$q_{s,0} = \dfrac{5431737.25}{2 \times 456036.7}$ $\left\{ \because q_{s,0} = \dfrac{T}{2A} \right\}$

$q_{s,0} = 5.955 \text{ N/mm}$

$q_{12} = 5.955$

$q_{23} = -30.29 + 5.955 = -24.33 \text{ N/mm}$

$q_{34} = -53.48 + 5.955 = -47.525 \text{ N/mm}$

$q_{45} = -66.02 + 5.955 = -60.065 \text{ N/mm}$

$q_{56} = -66.02 + 5.955 = -60.065 \text{ N/mm}$

$q_{67} = -53.48 + 5.955 = -47.525 \text{ N/mm}$

$q_{78} = -30.29 + 5.955 = -24.33 \text{ N/mm}$

$q_{89} = 5.955 \text{ N/mm}$

$q_{910} = 32.7 + 5.955 = 38.655 \text{ N/mm}$

$q_{1011} = 63 + 5.955 = 68.955 \text{ N/mm}$

$q_{1112} = 86.19 + 5.955 = 92.145 \text{ N/mm}$

$q_{1213} = 98.7 + 5.955 = 104.665 \text{ N/mm}$

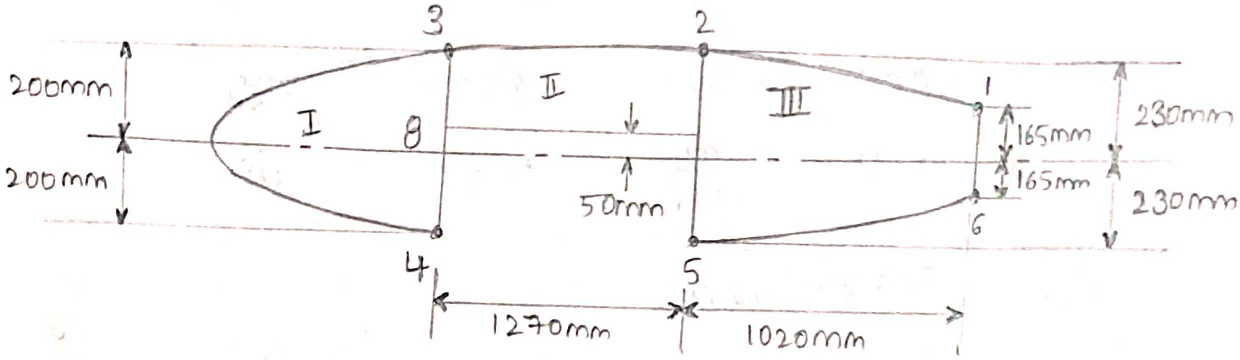
$q_{1314} = 98.7 + 5.955 = 104.665 \text{ N/mm}$

$q_{1415} = 85.9 + 5.955 = 91.855 \text{ N/mm}$

$q_{1516} = 62.7 + 5.955 = 68.6 \text{ N/mm}$

$q_{161} = 31.7 + 5.955 = 37.655 \text{ N/mm}$.

wing Section shown in fig has been idealized such that the booms Carry all the direct stresses. If the wing section is subjected to a bending moment of 300 kNm applied in a vertical plane, Calculate the direct stresses in the booms.



Boom areas:- $B_1 = B_6 = 2580 mm^2$ $\quad B_2 = B_5 = 3880 mm^2$

$B_3 = B_4 = 3230 mm^2$

Sol Given data:

$M_y = 0$

$M_x = 300 kNm = 300 \times 10^6 Nmm$

$I_{xy} = 0$ [$\because$ distribution of boom areas is symmetrical about x-axis]

$$\sigma = \frac{M_x}{I_{xx}} y$$

$$\sigma = \frac{300 \times 10^6}{I_{xx}} y$$

Booms	area	y	$Ay \times 10^3$	$I_{xx} \times 10^6$
1	2580	165	425.7	70
2	3880	230	892.4	205
3	3230	200	646	129
4	3230	-200	-646	129
5	3880	-230	-892.4	205
6	2580	~~150~~ -165	-425.7	70

$I_{xx} = 808 \times 10^6 \text{ mm}^4$

$$\sigma = \frac{M_x}{I_{xx}} \times y = \frac{300 \times 10^6}{808 \times 10^6} y = 0.371y$$

$\sigma_1 = 0.371y_1 = 61.2 \text{ N/mm}^2 \quad (y_1 = 165)$

$\sigma_2 = 0.371y_2 = 85.33 \text{ N/mm}^2 \quad (y_2 = 230)$

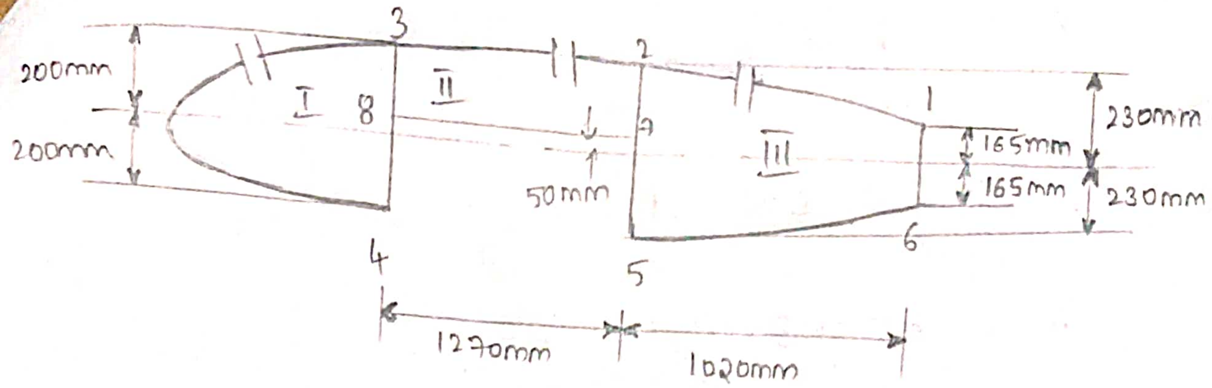
$\sigma_3 = 0.371y_3 = 74.2 \text{ N/mm}^2 \quad (y_3 = 200)$

$\sigma_4 = 0.371y_4 = -74.2 \text{ N/mm}^2 \quad (y_4 = -200)$

$\sigma_5 = 0.371y_5 = -85.33 \text{ N/mm}^2 \quad (y_5 = -230)$

$\sigma_6 = 0.371y_6 = -61.2 \text{ N/mm}^2 \quad (y_6 = -165)$

Problem 2 The wing section shown in Fig carries a vertically upward shear load of 86.8kN in a plane of web 572. The section has been idealised such that the booms resists all the direct stresses while the walls are effective only in shear. If the shear modulus of the wall is 27600N/mm². Calculate the shear flow distribution the section and the rate of twist.



wall	length (mm)	thickness (mm)	Cell area (mm^2)
1-2, 5-6	1023	1.22	$A_I = 265000$
2-3	1274	1.63	$A_{II} = 213000$
3-4	2200	2.03	$A_{III} = 413000$
4-8-3	400	2.64	
5-7-2	460	2.64	
6-1	330	1.63	
7-8	1270	1.22	

Boom Areas, $B_1 = B_6 = 2580\ mm^2$, $B_2 = B_5 = 3880\ mm^2$

$B_3 = B_4 = 3230\ mm^2$

Sol:- Given data:-

$S_x = 0$, $S_y = 86.8\ kN$

$I_{xy} = 0$

Booms	Area	y	$I_{xx} = Ay^2$ $\times 10^6$
1	2580	165	70
2	3880	230	205
3	3230	200	129
4	3230	-200	129
5	3880	-230	205
6	2580	-165	70

$I_{xx} = 808 \times 10^6 \ mm^4$

$$q_b = \frac{-S_y}{I_{xx}} A_i y_i$$

$\because q_b = 0$ at each cut $q_{12} = q_{23} = q_{34} = 0$

$$q_{b,16} = \frac{-86.8 \times 10^3}{808 \times 10^6} \times 2580 \times 165$$

$$q_{b,16} = -45.6 \ N/mm$$

$$q_{b,65} = \frac{-86.8 \times 10^3}{808 \times 10^6} \times 2580 \times (-165) + q_{b,16}$$

$$q_{b,65} = 45.6 - 45.6 = 0 \ N/mm$$

$$q_{b,57} = \frac{-86.8 \times 10^3}{808 \times 10^6} \times 3880 \times (-230) + 0$$

$$q_{b,57} = 95.86 \ N/mm$$

$$q_{b,27} = \frac{-86.8 \times 10^3}{808 \times 10^6} \times 3880 \times 230$$

$$q_{b,27} = -95.86 \ N/mm$$

$$q_{b,38} = \frac{-86.8 \times 10^3}{808 \times 10^6} \times 3230 \times 200$$

$$q_{b,38} = -69.39 \ N/mm$$

$$q_{b,48} = \frac{-86.8 \times 10^3}{808 \times 10^6} \times 3230 \times (-200)$$

$$= 69.39 \ N/mm$$

$$\theta = \frac{1}{2AG} \cdot \frac{q}{t} ds$$

for section - I

$$\theta_1 = \frac{1}{2 \times 265000 \times 27600}\left[\frac{q_{s,0,I}}{2.03} \times 2200 + \frac{q_{s,0,I}}{2.64} \times 200 + \frac{q_{sI} - q_{sII}}{2.64} \times 150\right] \quad (1)$$

θ for II

$$\theta_2 = \frac{1}{2 \times 213000 \times 27600}\left[\frac{q_{s0,II} - q_{s0,I}}{2.64} \times 150 + \frac{q_{s0,II}}{1.63} \times 1274 + \frac{q_{s0,II} - q_{s0,III}}{2.64} \times 180 + \frac{q_{s0,II}}{1.22} \times 1270\right] \quad (2)$$

θ for III

$$\theta_3 = \frac{1}{2 \times 413000 \times 27600}\left[\frac{q_{s0,III} - q_{s0,II}}{2.64} \times 180 + \frac{q_{s0,III}}{2.64} \times 280 + \frac{q_{s0,III}}{1.63} \times 330 + \frac{q_{s0,III}}{1.22} \times 1023\right] \quad (3)$$

Now taking moments about 7

$$T = -q_{38} \times 150 \times 1270 + (-q_{48} \times 250 \times 1270) + q_{16} \times 330 \times 1020$$

$$= -24.1 \times 10^6$$

$$T = 2A_1 q_1 + 2A_2 q_2 + 2A_2 q_3$$

$$\therefore -24 \times 10^6 = 2\left[265000\, q_{s,0,I} + 213000\, q_{s,0,II} + 2 \times 413000\, q_{s,0,III}\right]$$

$$-24 \times 10^3 = 530\, q_{s,0,I} + 426\, q_{s,0,II} + 826\, q_{s,0,III} \quad (4)$$

~~After solving eq~~

~~$q_{s0I} = -21.47$~~

~~$q_{s0II} = 9.77$~~

~~$q_{s0III} = -20.31$~~

Let,

$\theta_1 = \theta_2$

$$7.96 \times 10^{-8} q_{s0I} + 16.3 \times 10^{-8} q_{s0II} - 0.57 \times 10^{-8} q_{s0III} \quad \text{--- (5)}$$

& $\theta_2 = \theta_3$

$$-0.48 \times 10^{-8} q_{s0I} + 16.4 \times 10^{-8} q_{s0II} + 8.43 \times 10^{-8} q_{s0III} \quad \text{--- (6)}$$

& we know that

$$T = -24 \times 10^3$$

Solving Eqn (4) (5) & (6)

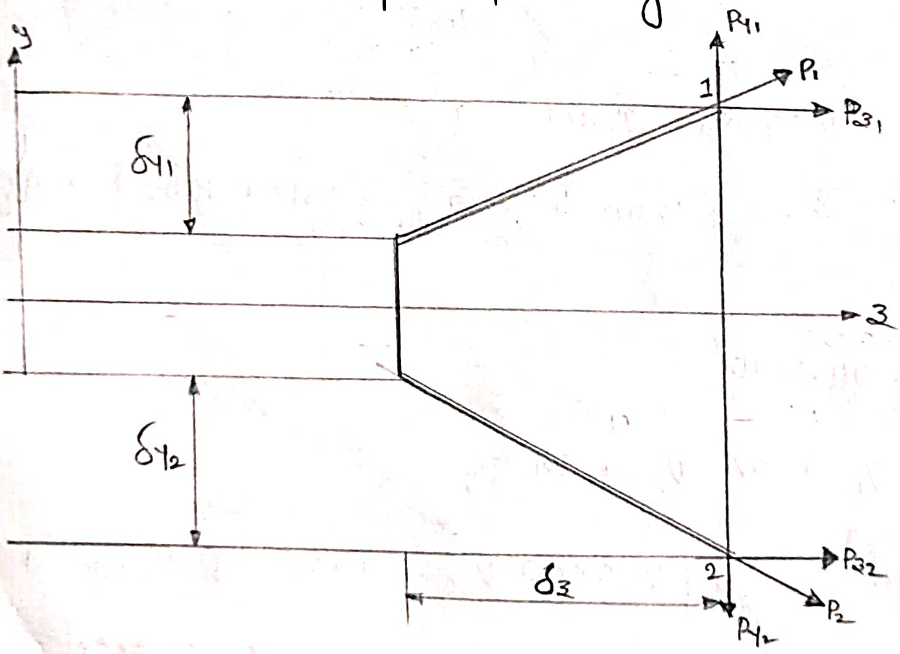
$$q_{s,0I} = -21.47$$

$$q_{s,0II} = 9.77$$

$$q_{s,0III} = -20.31$$

Add these values to the basic shear flow values

" Tapered wing "



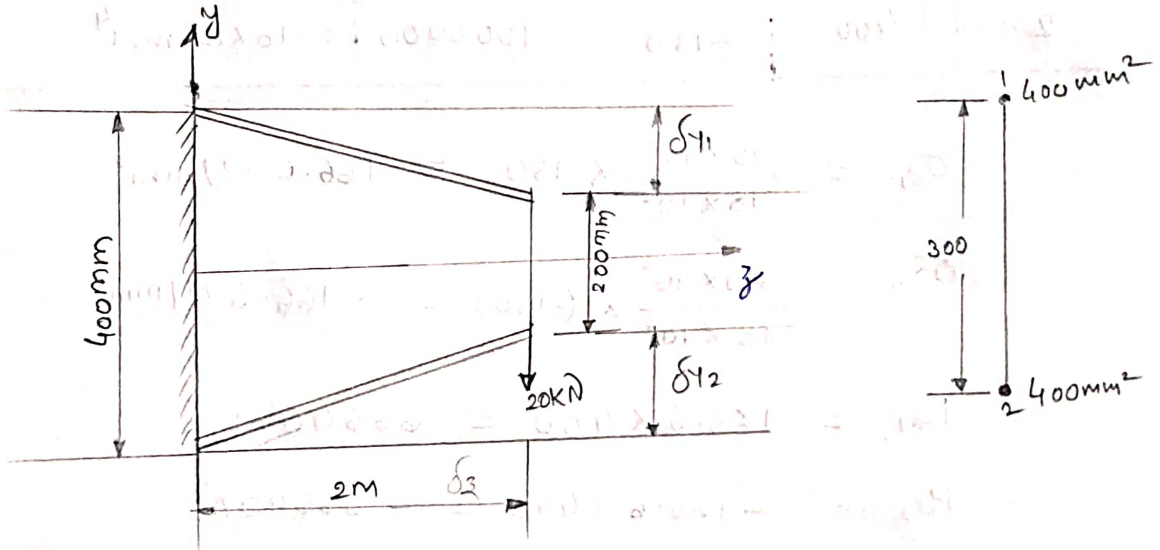
Pz1 and Pz2 are the components in the z direction of the axial loads P1 and P2 in the flanges and Py1 and Py2 are the components parallel to y axis.

$$S_y = S_{yw} - P_{z_1} P_{y_1} - P_{z_2} P_{y_2}$$

$$P_{z_1} = \sigma_{z_1} \times B_1 \quad , \quad P_{z_2} = \sigma_{z_2} \times B_2$$

$$P_{y_1} = \frac{\delta_{y_1}}{\delta_z} \quad , \quad P_{y_2} = \frac{\delta_{y_2}}{\delta_z}$$

Problem:- Determine the shear flow distribution in the web of the tapered beam shown in fig. at a section mid way along its length. the web of the beam has a thickness of 2mm & its is fully effective in resisting direct stress. the beam tapered symmetrically about its horizontal axis & cross-sectional area of the boom is 400mm². the internal bending moment & the shear load at the mid section is applied externaly.



Given Data :-

$S_{yw} = -20KN = -20 \times 10^3$

$S_z = 0$

$I_{zy} = 0$

$\delta y_1 = 100$

$\delta y_2 = -100$

$\delta z = 2000$

$$S_y = S_{yw} - P_{z_1} P_{y_1} - P_{z_2} P_{y_2}$$

$$q = -\frac{S_y}{I_{zz}} A_i y_i$$

$$P_{y_1} = \frac{\delta_{y_1}}{\delta_z} = \frac{100}{2000} = 0.05\,N$$

$$P_{y_2} = \frac{\delta_{y_2}}{\delta_z} = \frac{-100}{2000} = -0.05\,N$$

$$P_{z_1} = \sigma_{z_1} \times B_1 \quad , \quad P_{z_1} = \sigma_{z_2} \times B_2$$

$$\sigma_z = \frac{M_z}{I_{zz}} \cdot y$$

$$\left\{ \because M_y = 0,\ M_z = 20\times10^3\times10^3 \right.$$
$$M_z = 20\times10^6\ N/mm$$

Booms	Area	y	Ah^2	$I_{zz} = \Sigma Ah^2$
1	400	150	9000000	18000000
2	400	−150	9000000	$= 18\times10^6\ mm^4$

$$\sigma_{z_1} = \frac{20\times10^6}{18\times10^6}\times 150 = 166.6\ N/mm^2$$

$$\sigma_{z_2} = \frac{20\times10^6}{18\times10^6}\times(-150) = -166.6\ N/mm^2$$

$$\therefore\ P_{z_1} = 166.6\times400 = 66640\,N$$

$$P_{z_2} = -166.6\times400 = -66640\,N$$

$$\therefore\ S_y = -20\times10^3 - (666.40\times0.05) - (-66640\times(-0.05))$$

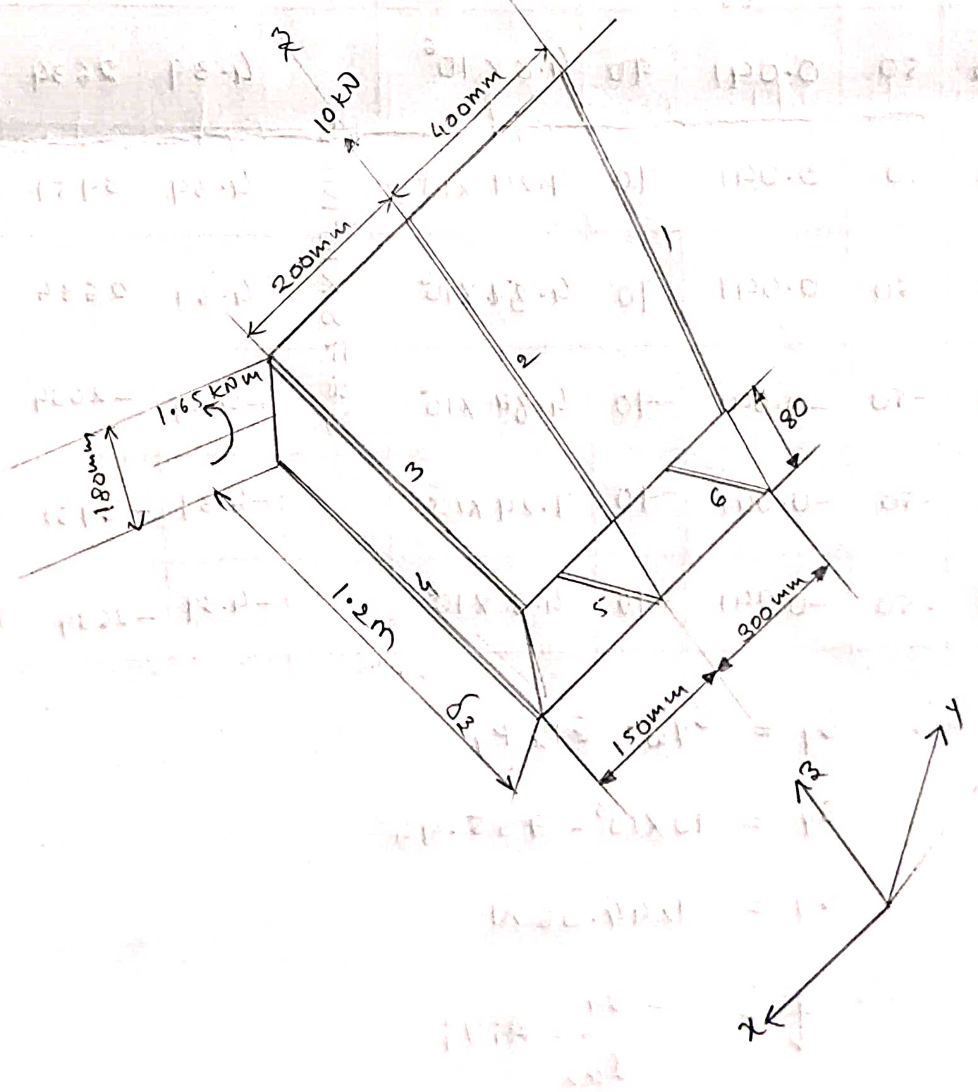
$$S_y = -26664\,N$$

$$q_{12} = \frac{-S_y}{I_{zz}} A_1 y_1$$

$$q_{12} = \frac{26664}{18\times10^6}\times400\times150$$

$$q_{12} = 88.8\ N/mm.$$

2 cell beam has singly symmetrical cross-sections, 1.2m a path & tapers symmetrically in the y- direction about a longitudinal axis. the beam supports load which produces a shear force of 10KN in y-direction & bending moment Mx of 1.65 KN-M at a larger cross-section the shear load is applied in the plane of internal spar web if the booms 1 & 6 lie in a plane which is parallel to Y3 plane, Calculate the force in the booms & then shear flow distribution in the walls at the larger x-section. the booms are assumed to resist all the direct stresses while the walls are effective only in shear the shear module is constant throughout the vertical-webs are 1mm thickness while the remaining wall are 0.8 mm thick.



Given :- Booms Area

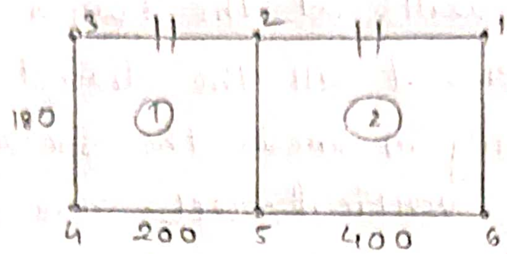
$B_1 = B_3 = B_4 = B_6 = 600\,mm^2$

$B_2 = B_5 = 900\,mm^2$

$S_{y\omega} = 10\,kN = 10 \times 10^3\,N$

$M_x = 1.65\,kN\text{-}M = 1.65 \times 10^6\,N\,mm$

$\delta_3 = 1.2\,m = 1200\,mm$



Booms	Area	σ_y	$P_y = \frac{\delta y}{\delta 3}$	y	$Ah^2 \times 10^6$	$I_{xx} = \Sigma Ah^2$	σ_x	$P_x = \sigma_x \times B$	$P_x \times P_y$	$\Sigma P_x P_y$
1	600	50	0.041	90	4.8×10^6		4.39	2634	107.99	
2	900	50	0.041	90	7.29×10^6		4.39	3951	161.99	
3	600	50	0.041	90	4.88×10^6	$33.78 \times 10^6\,mm^4$	4.39	2634	107.99	755.94
4	600	-50	-0.041	-90	4.89×10^6		-4.39	-2634	107.99	
5	900	-50	-0.041	-90	7.29×10^6		-4.39	-3951	161.99	
6	600	-50	-0.041	-90	4.8×10^6		-4.39	-2634	107.99	

$\therefore\ S_y = S_{y\omega} - \Sigma P_x P_y$

$S_y = 10 \times 10^3 - 755.94$

$S_y = 9244.06\,N$

$\therefore\ q = \frac{-S_y}{I_{xx}} A_i y_i$

$$q_{b34} = \frac{-9244.06}{33.78\times10^6}\times 600\times 90 = -14.77 \text{ N/mm}$$

$$q_{b25} = \frac{-9244.06}{33.78\times10^6}\times 900\times 90 = -22.16 \text{ N/mm}$$

$$q_{b16} = \frac{-9244.06}{33.78\times10^6}\times 600\times 90 = -14.77 \text{ N/mm}$$

$$q_{b45} = q_{b65} = q_{b32} = q_{b21} = 0$$

Angle of twist for cell ①

$$\theta = \frac{1}{2AG}\frac{q_{s0I}}{t}.ds$$

$$\theta_1 = \frac{1}{2\times36000\times G}\times\left[\frac{q_{s0I}}{0.8}\times200 + \frac{q_{s0I}}{1}\times180 + \frac{q_{s0I}}{0.8}\times200 + \frac{q_{s0I}-q_{s0II}}{1}\times180\right]$$

$$\theta_1 = 11.9\times10^{-3}q_{s0I} - 2.5\times10^{-3}q_{s0II} \quad \text{——} \; (1)$$

Angle of twist for cell ②

$$\theta_2 = \frac{1}{2\times72000\times G}\times\left[\frac{q_{s0II}}{0.8}\times400 + \frac{q_{s0II}-q_{s0I}}{1}\times180 + \frac{q_{s0II}}{0.8}\times400 + \frac{q_{s0II}}{1}\times180\right]$$

$$\theta_2 = -1.25\times10^{-3}q_{s0I} + 9.44\times10^{-3}q_{s0II} \quad \text{——} \; (2)$$

$$\theta_1 = \theta_2$$

$$0.013q_{s0I} - 0.0119q_{s0II} \quad \text{——} \; (3)$$

Applying the bredt-batho theory

$$T = 2A_1q_{s0I} + 2A_2q_{s0II}$$

to find T let us take moment w.r.to 2-5

$$T = -q_{34}\times180\times200 + q_{16}\times180\times400$$

$$T = -531720 \text{ N-mm}$$

$\therefore \quad -531720 = 2 \times 360000 q_{soI} + 2 \times 720000 q_{soII}$

$720000 q_{soI} + 144000 q_{soII} = -531720$ —— (4)

solving Eq (3) & (4)

$q_{soI} = -2.31 \ N/mm$

$q_{soII} = -2.53 N/mm$

$\therefore$

$q_{23} = q_{b23} + q_{soI} = 0 + (-2.31) = -2.31 \ N/m$

$q_{34} = q_{b34} + q_{soI} = -14.77 - 2.31 = -17.08 \ N/mm$

$q_{45} = q_{b45} + q_{soI} = 0 - 2.31 = -2.31 N/mm$

$q_{25} = q_{b25} + q_{soI} + q_{soII} = -22.16 - 2.31 - 2.53$

$q_{25} = -27 \ N/mm$

$q_{21} = q_{b21} + q_{soII} = 0 - 2.53 = -2.53 \ N/mm$

$q_{16} = q_{b16} + q_{soII} = -14.77 - 2.53 = 17.3 \ N/mm$

$q_{65} = q_{b65} + q_{soII} = 0 - 2.53 = -2.53 N/mm$

Unit V

Energy Methods:

strain energy due to axial, bending and Torsional loads – Castigliano's theorem

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Strain energy (or) Resilience:

When the elastic body is loaded it undergoes deformation i.e. its dimensions change and when it is relieved of the load it regains its original shape. (At the same time, the energy stored in the elastic body is called as strain energy). For the time loaded energy is stored in it, the same is given up (or) released by the loading when the load is removed. This energy is Called as strain energy. The ~~sto~~ strain energy stored "within" the elastic limit, when loaded externally is Called "Resilience", and the maximum ~~energ~~ energy which a body stores "upto" elastic limit is Called "proof resilience".

Resilience:

The strain energy stored "within" the elastic limit, when loaded externally is Called as "Resilience".

Proof Resilience:

The maximum energy stored

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within a body "upto" elastic limit is Called as "Proof resilience". ③

[Proof resilience is the mechanical property of materials and it indicates their Capacity to bear shocks.

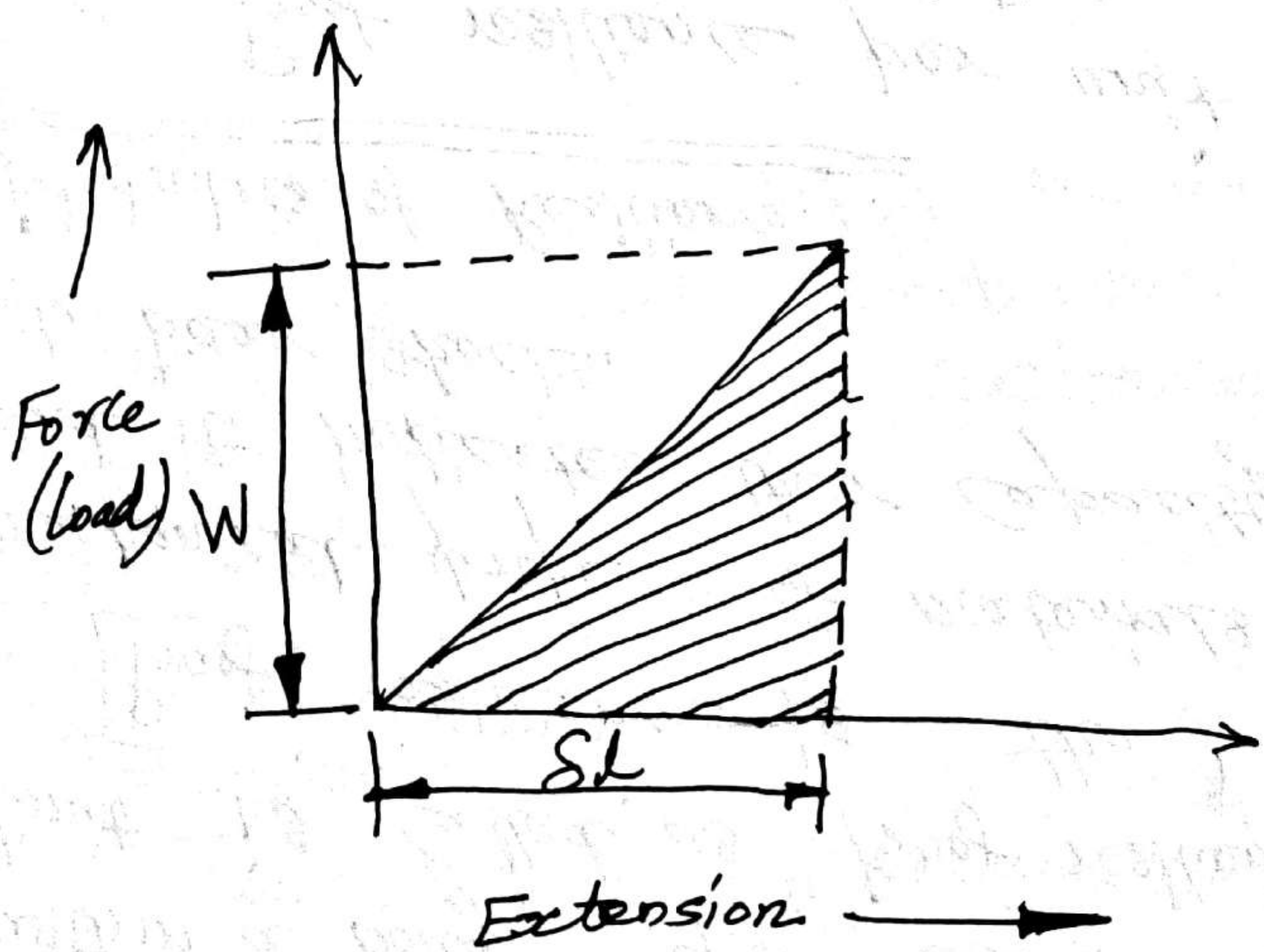
Modulus of Resilience:

Proof resilience per unit Volume of piece is Called "Modulus of resilience".

Strain energy in Simple tension and Compression: (Strain energy due to axial load)

Let us take the Case of a bar of Cross-Sectional area "A" and length "L" and Subjected to a load "W". Suppose this load extends the bar by an amount δl and produces a maximum ~~stress~~ stress, σ.

The Workdone by W and hence the strain energy (U) stored in the material is equal to the area under the force - extension curve.



Strain energy stored in the bar = Work done by the load.

$$U = \tfrac{1}{2} \cdot W \cdot \delta l$$

$$U = \frac{W}{2} \times \frac{\sigma l}{E} \qquad \left[\because \delta l = \frac{\sigma l}{E}\right.$$

$$U = \frac{\sigma A}{2} \times \frac{\sigma l}{E} \qquad \boxed{W = \sigma \times A}$$

$$\boxed{V = Al}$$

$$U = \frac{\sigma^2 Al}{2E} = \frac{\sigma^2 V}{2E}$$

Where, $A \Rightarrow$ Area

$E \Rightarrow$ Modulus of Elasticity

$l \Rightarrow$ length of bar

$V \Rightarrow$ Volume of bar

$\sigma \Rightarrow$ stress.

If σ_p be the proof stress

(or) the maximum stress to which the bar is stressed upto the elastic limit, then

Proof resilience,

$$\boxed{U_p = \frac{\sigma_p^2}{2E} \times V}$$

and, modulus of resilience, $= \dfrac{\sigma_p^2}{2E}$

5.1) A ~~bar of~~ steel bar of 4cm by 4cm in Section, 3m long is subjected to an axial pull of 128 KN. Taking $E = 200\ GN/m^2$. Find the alternation in the length of the bar. Calculate also the amount of energy stored in the bar during the extension.

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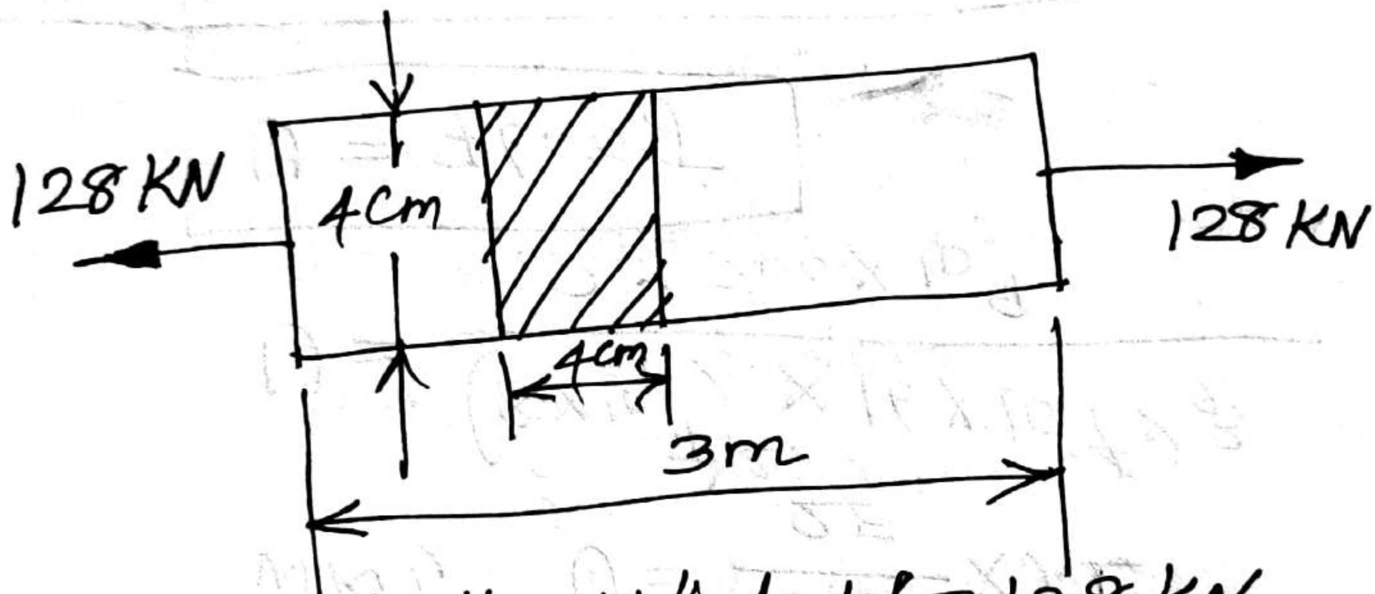
Solution:

Cross sectional area of the bar, $A = 4\,cm \times 4\,cm = 16\,cm^2$

$$\boxed{A = 16 \times 10^{-4}\,m^2}$$



Axial pull applied, $W = 128\,KN$

Length of the bar $= l = 3m.$

Modulus of elasticity,

$$E = 200\,GN/m^2$$

$$E = 200 \times 10^9\,N/m^2.$$

Elongation of bar, δl:—

$$\delta l = \frac{Wl}{AE}$$

$$\delta l = \frac{128 \times 10^3 \times 3}{16 \times 10^{-4} \times 200 \times 10^9}$$

$$\delta l = 0.0012\, m \text{ (or) } \underline{1.2\, mm}$$

Energy stored in the bar during elongation, U:-

$$\sigma = \frac{W}{A} = \frac{128 \times 10^3}{16 \times 10^{-4}}$$

$$\sigma = 16 \times 10^7\, N/m^2$$

WKT, $U = \frac{\sigma^2}{2E} \times A L$

$$U = \frac{(8 \times 10^7)^2 \times 16 \times 10^{-4} \times 3}{2 \times 200 \times 10^9}$$

$$\boxed{U = 76.8\, J}$$

(5.2) A steel specimen $1.5\, cm^2$ in cross-section stretches 0.05mm over 5cm gauge length under an axial load of 30KN. Calculate the strain energy

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stored in the specimen (9)
at this point. If the load at the elastic limit for specimen is 50 KN. Calculate the elongation at the elastic limit and the resilience.

Solution:

Cross-Sectional area of

specimen, $A = 1.5\ cm^2$

$A = 1.5 \times 10^{-4}\ m^2$.

Increase in length over 5 cm gauge length, $\delta l = 0.05\ mm$

$\delta l = 0.05 \times 10^{-3}\ m$

Axial load, $W = 30\ KN = 30 \times 10^3\ N$

Load at elastic limit $= 50\ KN$.

$= 50 \times 10^3\ N$.

~~stop in~~ strain energy stored in specimen, U:-

$$U = \frac{\sigma^2 A l}{2E} = \frac{1}{2} \cdot W \cdot \delta l$$

$$U = \frac{1}{2} \times (30 \times 10^3) \times 0.05 \times 10^{-3}$$

$$\boxed{U = 0.75 J}$$

Also, $\boxed{E = \frac{W}{A} \times \frac{l}{\delta l}}$

$$E = \frac{\text{stress}}{\text{strain}} = \frac{W/A}{\delta l / l}$$

$$E = \frac{W \cdot l}{A \cdot \delta l}$$

$$E = \frac{30 \times 10^3 \times (5/100)}{1.5 \times 10^{-4} \times 0.05 \times 10^{-3}}$$

$$E = 200 \times 10^9 \ N/m^2$$

Elongation at elastic limit δl:

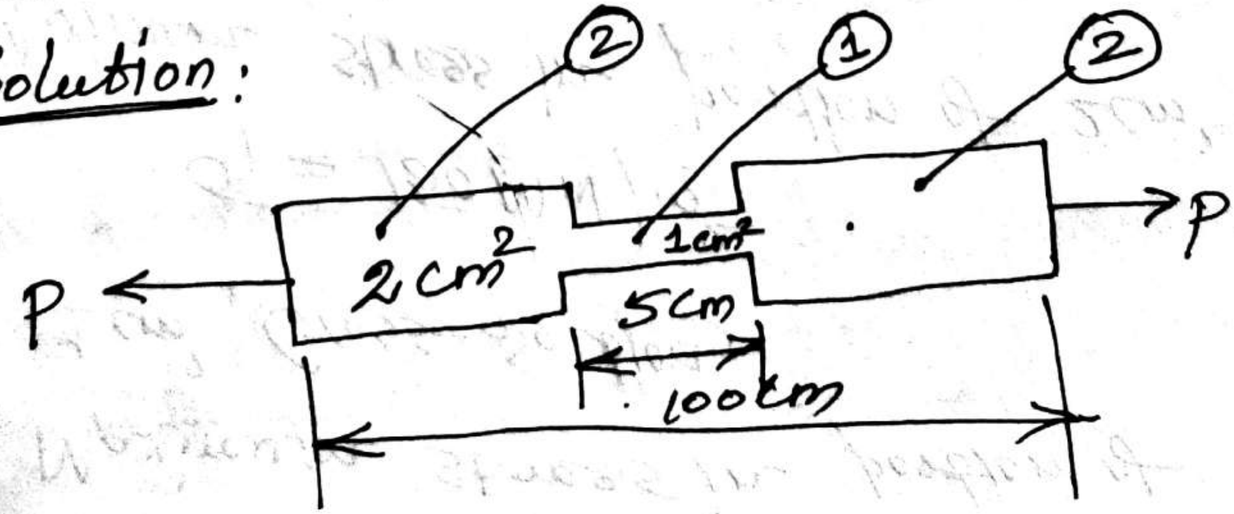
$$\delta l = \frac{Wl}{AE} = \frac{50 \times 10^3 \times (5/100)}{(1.5 \times 10^{-4}) \times 200 \times 10^9}$$

$\delta l = 0.000083.3 m$

$$\boxed{\delta l = 0.0833 mm}$$

(5.3) A bar 100 cm in length is subjected to an axial pull, such that the maximum stress is equal to 150 MN/m^2. Its area of Cross-Section is 2 cm^2 over a length of 95 cm and for the middle 5 cm length it is only 1 cm^2. If E = 200 GN/m^2, Calculate the strain energy stored in the bar.

<u>Solution:</u>



Maximum stress in portion of $1\,cm^2$ Cross-Section,

$$\sigma_1 = 150\,MN/m^2$$

Maximum stress in portion of $2\,cm^2$ Cross-section,

$$\sigma_2 = 75\,MN/m^2.$$

WKT, $\sigma_1 A_1 = \sigma_2 A_2$

$$150 \times 1 \times 10^{-4} = \sigma_2 \times 2 \times 10^{-4}$$

$$\boxed{\sigma_2 = \frac{150}{2} = 75\,MN/m^2}$$

strain energy stored in the bar,

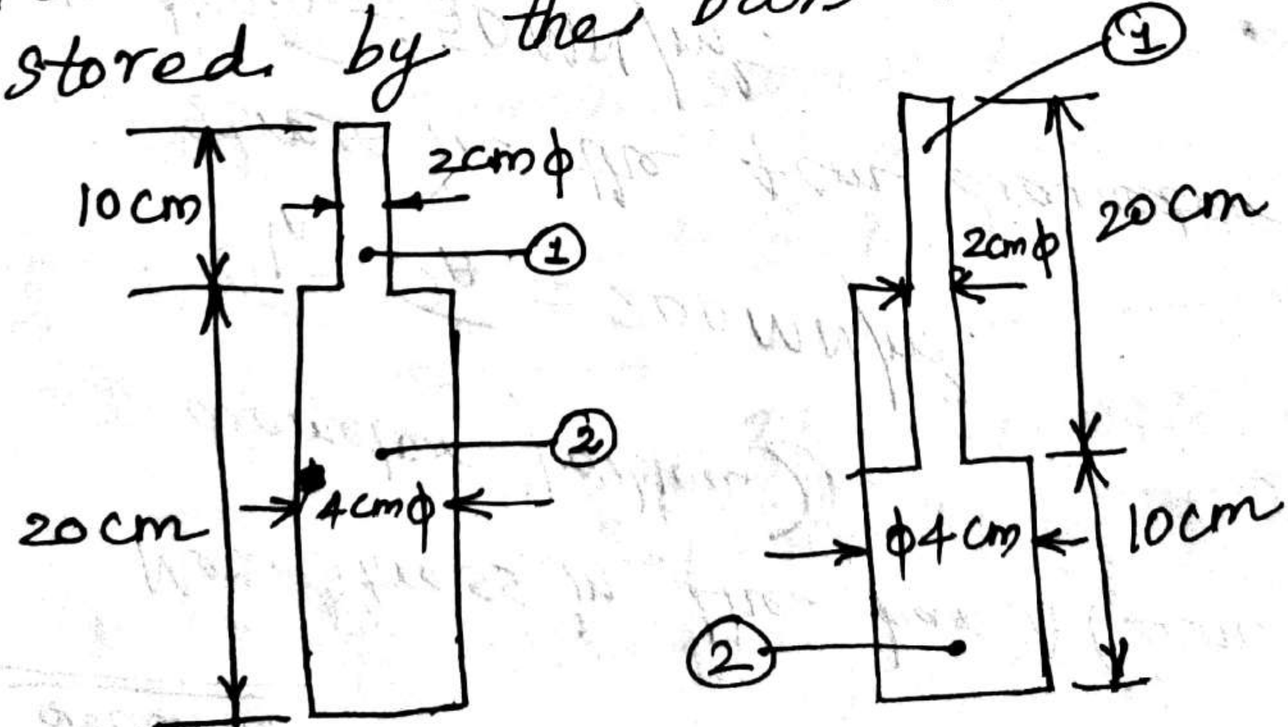
$$U = \frac{\sigma_1^2 A_1 l_1}{2E} + \frac{\sigma_2^2 A_2 l_2}{2E}$$

$$U = \frac{(150 \times 10^6)^2 \times (1 \cdot \times 10^{-4}) \times (5/100)}{2 \times (200 \times 10^9)}$$

$$+ \frac{(75 \times 10^6)^2 \times (2 \times 10^{-4}) \times (95/100)}{2 \times (200 \times 10^9)}$$

$U = \underline{\underline{2.953 Nm \text{ (or) Joules.}}}$

(5.4) Two similar bars A and B are each 30cm long as shown in figure. The bar A receives an axial blow, which produces a ~~max.~~ maximum stress of 200 MN/m². Find ~~maxi~~ max. stress produced by the same blow on the bar B. If the bar is stressed to 200 MN/m². Determine the ratio of energy stored by the bars and A and B.



Solution:

Max. stress in the bar A (2cm diameter portion),

$\sigma_{1_A} = \sigma_A = 200 MN/m^2$.

$\therefore$ stress in the 4cm diameter portion $\sigma_{2A} = 50 MN/m^2$.

$\therefore \sigma_1 A_1 = \sigma_2 A_2$

$$200 \times \frac{\pi}{4}\left(\frac{2}{100}\right)^2 = \sigma_2 \times \frac{\pi}{4} \times \left(\frac{4}{100}\right)^2$$

$$\sigma_2 = \sigma_{2A} = \frac{200}{4} = 50 MN/m^2$$

Max. stress in bar B (2cm diameter ~~por~~ portion), $\sigma_{B1} = ?$

Stress in 4cm diameter portion,

$$\sigma_{B2} = \frac{\sigma_{B1}}{4}$$

(according to previous result) 15

~~Energy~~ Energy stress in bar A,

$$U_A = \frac{\sigma_1^2 A_1 l_1}{2E} + \frac{\sigma_2^2 A_2 l_2}{2E}$$

$$U_A = \frac{(200 \times 10^6)^2 \times \frac{\pi}{4} \times \left(\frac{2}{100}\right)^2 \times \left(\frac{10}{100}\right)}{2 \times E} + \frac{(50 \times 10^6)^2 \times \frac{\pi}{4} \times \left[\frac{4}{100}\right)^2 \times \frac{20}{100}}{2 \times E}$$

$$U_A = \frac{2\pi \times 10^{11}}{E} + \frac{\pi \times 10^{11}}{E} = \frac{3\pi \times 10^{11}}{E} \quad (1)$$

Similarly, Energy stored by the bar B,

$$U_B = \frac{\sigma_B^2 \times \frac{\pi}{4} \times \left(\frac{2}{100}\right)^2 \times \left(\frac{20}{100}\right)}{2E} + \frac{\left(\frac{\sigma_B}{4}\right)^2 \times \frac{\pi}{4} \times \left(\frac{4}{100}\right)^2 \times \left(\frac{10}{100}\right)}{2E}$$

$$U_B = \frac{2.25 \pi \sigma_B^2}{10^5 \times 2E} \quad \text{--- (2)}$$

Since the blow on the bars A and B is the Same, therefore the two energies are equal.

$$(1) = (2)$$

$$\frac{3\pi \times 10^{11}}{E} = \frac{2.25 \pi \times \sigma_B^2}{10^5 \times 2E}$$

$$\sigma_B^2 = \frac{3 \times 10^{11} \times 10^5 \times 2}{2.25} = 1.633 \times 10^8 \ N/m^2.$$

$$\therefore \sigma_B = 163.3 \ MN/m^2$$

Ratio of energy stored by bars A and B: $\left(\frac{U_A}{U_B}\right)$:—

Energy stored in bar B when it is also stressed to $200 MN/m^2$.

$$U_B = \frac{2.25 \pi \sigma_B^2}{10^5 \times 2E}$$

$$U_B = \frac{2.25\pi \times (200 \times 10^6)^2}{10^5 \times 2E}$$

$$U_B = \frac{4.5\pi \times 10^{11}}{E}$$

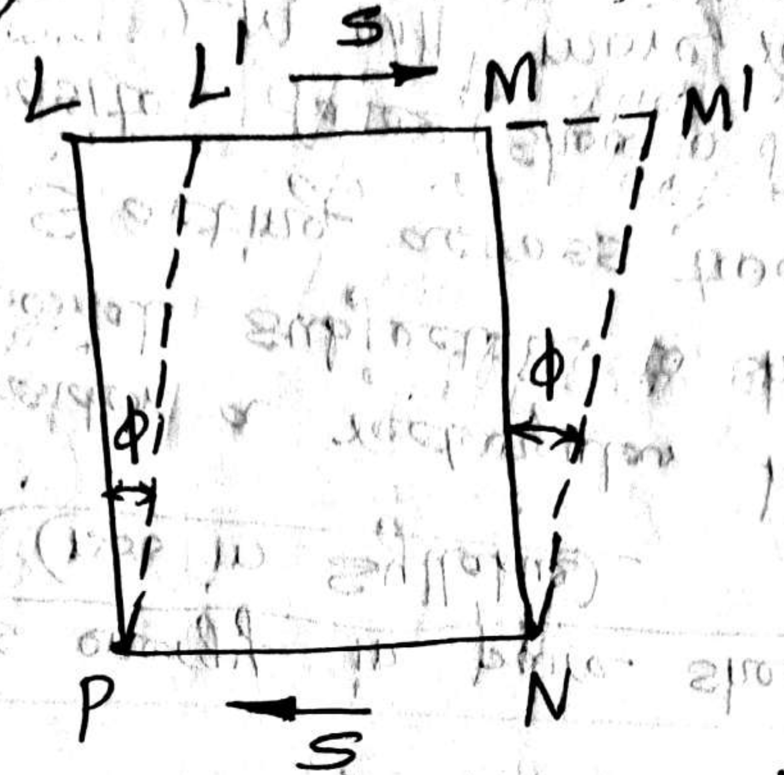
$\therefore$ Ratio of energy stored by A and B :

$$\frac{U_A}{U_B} = \frac{\frac{3\pi \times 10^{11}}{E}}{\frac{4.5\pi \times 10^{11}}{E}} = 2/3.$$

Stress energy in pure shearing

Consider a rectangular block of material subjected to shearing forces S acting across two of its opposite faces (show in figure). The face LM will more, relative to face NP, a distance ~~MM'~~

$MM' = MN \times \phi$, where ϕ is the angle of shear produced.



The workdone $= \frac{1}{2} S \times MM'$

$$= \frac{1}{2} \times S \times MN \times \phi$$

$\left[\because \frac{MM'}{MN} = \tan\phi = \phi\right.$, because ϕ is Very Small$\left.\right]$

Now, $S = T \times LM$, where T is the shearing stress, and $\phi = \frac{T}{C}$

($\phi = e_s$ = Shear strain)

Taking Unit depth normal to diagram, we have

Strain energy = Workdone

$$= \frac{1}{2} \tau \times LM \times MN \times \frac{\tau}{C}$$

$$= \frac{1}{2} \times \frac{\tau^2}{C} \times LM \times MN$$

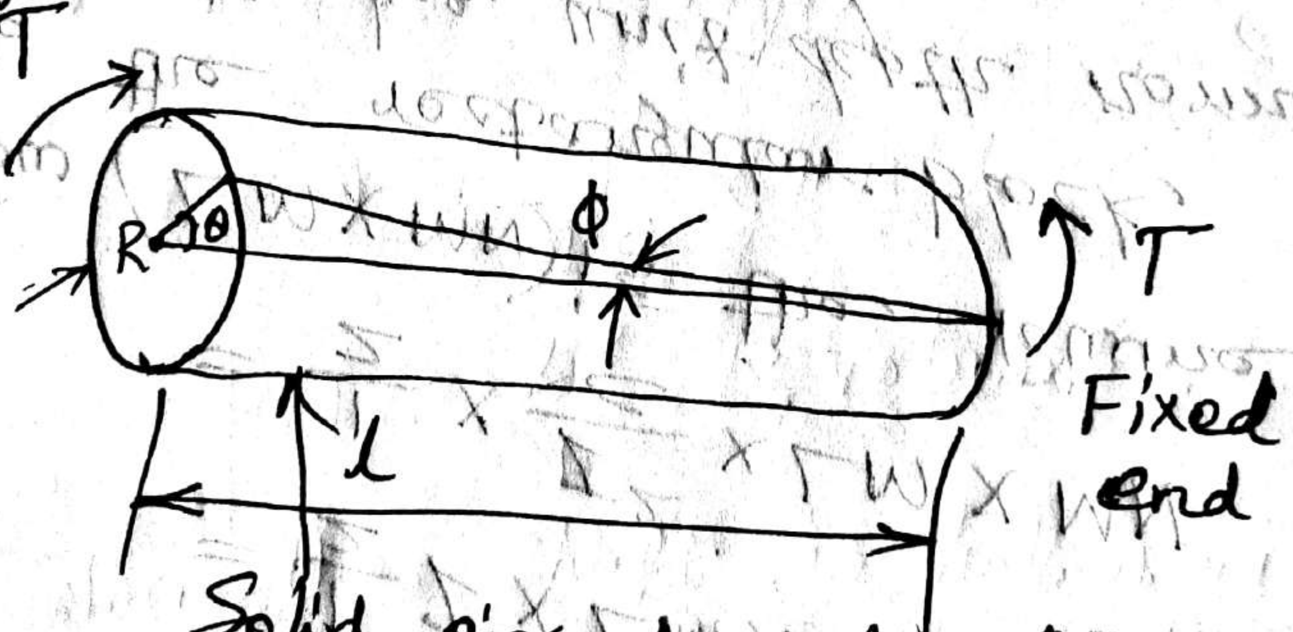
Now $(LM \times MN)$ is the Volume of the rectangular block, since it has unit depth normal to LMNP.

$\therefore$ Strain energy, $U = \frac{\tau^2}{2C} \times$ Volume of block

This is the shearing strain energy for a block of material sub. to a Const. shearing stress throughout.

Strain energy in Torsion:

Consider a Solid Circular shaft of length l and radius R, subjected to a torque T producing a twist θ in the length of the shaft (Shown in ~~fig~~ figure].



Solid circular shaft

The workdone $= \frac{1}{2} T\theta$, which is stored in the shaft as strain energy.

But, $\frac{T}{I_p} = \frac{C\theta}{l} = \frac{\tau}{R}$

(or)

$$\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{R}$$

Where, T = Torque applied

I_p (or) J = Polar moment of Inertia

C = Modulus of rigidity

l = Length of the shaft,

τ = Maximum shear stress on the surface on the shaft.

$$T = \frac{\tau \times J}{R} \quad \text{and} \quad \theta = \frac{\tau l}{CR}$$

$$\therefore \text{Workdone} = \frac{1}{2} \times \frac{\tau \times J}{R} \times \frac{\tau \times l}{CR}$$

$$\therefore \text{Workdone} = \frac{1}{2} \times \frac{\tau^2}{C} \times \frac{J l}{R^2}$$

Now, $J = \dfrac{\pi R^4}{2}$

$$\therefore \text{Workdone} = \frac{1}{2} \times \frac{\tau^2}{C} \times \frac{\pi R^4 \times l}{2 R^2}$$

$$\text{Workdone} = \frac{1}{4} \times \frac{\tau^2}{C} \times \pi R^2 l$$

(or) strain energy, $\boxed{\text{Volume} = \pi R^2 l}$

$$U = \frac{\tau^2}{4C} \times \pi R^2 l$$

$$\boxed{U = \frac{\tau^2}{4C} \times \text{Volume}}$$

When the shaft is hollow with an external radius R and internal radius r:

Again, Workdone $= \frac{1}{2} T\theta$

and $\theta = \dfrac{\tau l}{CR}$ and

$$T = \frac{IJ}{R}$$

$$\therefore \text{Workdone} = \frac{I^2}{2C} \times \frac{Jl}{R^2}$$

But, $J = \frac{\pi}{2}(R^4 - r^4)$

$$\text{So, Workdone} = \frac{I^2}{2C} \times \frac{\pi l (R^4 - r^4)}{2R^2}$$

$$= \frac{I^2}{4C} \times \frac{\pi l (R^2 + r^2)(R^2 - r^2)}{R^2}$$

$$= \frac{I^2}{4C} \times \frac{\pi l (R^2 + r^2)(R^2 - r^2)}{R^2}$$

$$= \frac{I^2}{4C} \times \frac{(R^2 + r^2)}{R^2} \times \pi (R^2 - r^2) l$$

$$\boxed{\therefore \text{strain energy} = \frac{I^2}{4C} \times \frac{(R^2 + r^2)}{R^2} \times V,}$$

where $V = \text{Volume} = \pi (R^2 - r^2) l.$

(5.5) The external diameter of a hollow shaft is twice the internal diameter. It is sub. to pure torque and it attains a maximum shear stress τ. Show that the strain energy stored per unit volume of the shaft is $\frac{5\tau^2}{16C}$. Such a shaft is required to transmit 5400 KW at 110 rpm, with uniform torque, the max. stress not exceeding 84 MN/m². Find:

(i) The shaft diameters

(ii) The energy stored per m³.

Take : $C = 90\, GN/m^2$.

~~Given data:~~ Solution:

Let R = External radius

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of hollow shaft, and

r = Internal radius of the hollow shaft = $R/2$ (Given).

~~Then~~ The General formula for strain energy (hollow Shaft):

$$U/\text{volume} = \frac{I^2}{4C} \times \frac{(R^2 + r^2)}{R^2} \cancel{\times \pi (R^2 - r^2) \times L}$$

Sub. $r = R/2$;

$$U/\text{Volume} = \frac{I^2}{4C} \times \frac{R^2 + \frac{R^2}{2^2}}{R^2}$$

$$U/\text{Volume} = \frac{I^2}{4C} \times \frac{4R^2 + R^2}{4R^2}$$

$$U/\text{Volume} = \frac{5I^2 R^2}{16CR^2}$$

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$$\boxed{U/\text{Volume} = \frac{5\tau^2}{16C}}.$$

Power required to be transmitted,

$P = 5400\,kW = 54 \times 10^5$ Watts.

Speed, $N = 110$ rpm.

Max. Shear stress, $\tau = 84\,MN/m^2$

$\tau = 84 \times 10^6\,N/m^2$.

(i) The shaft diameter, D:—

Now, $P = \frac{2\pi NT}{60}$

$$54 \times 10^6 = \frac{2\pi \times 110\,T}{60}$$

$$\boxed{T = 468783\,Nm}$$

~~Also,~~ Also, $\frac{T}{J} = \frac{\tau}{R}$

$$T = \frac{I}{R} \times \tau \quad \cancel{\frac{I}{D/2} \times \frac{\pi}{32}(D^4 - d^4)}$$

$$T = \frac{I}{D/2} \times \frac{\pi}{32}(D^4 - d^4)$$

$$468783 = \frac{\pi}{16}\left[\frac{D^4 - d^4}{D}\right] \times I$$

$$468783 = \frac{\pi}{16}\left[\frac{D^4 - \left(\frac{D}{2}\right)^4}{D}\right] \times I$$

$$468783 = \frac{\pi}{16} \times \frac{16D^4 - D^4}{D} \times I$$

$$468783 = \frac{\pi}{16} \times \frac{15D^4}{D} \times I$$

(I is given) $\cancel{I}$

$$468783 = \frac{\pi}{16} \times 15D^3 \times 84 \times 10^6$$

$$D^3 = 30322.3 \times 10^{-6}$$

$D = 0.312\ m$

$$\boxed{D = 312\ mm}$$

$d = D/2 = \frac{312}{2} = \underline{156 mm.}$

(ii) Energy stored per m^3:

$$U/\text{volume} = \frac{5\tau^2}{16C} = \frac{5}{16} \times \frac{(84\times10^6)^2}{90\times10^9}$$

$$U/\text{volume} = 24500\ J/m^3$$

Hence, energy stored per m^3

$= \underline{24.5\ KJ/m^3.}$

(5.6) Compare the ~~stre~~ strain energies of the following two shafts sub. to ~~the~~ the same maximum shear stress in torsion:

(i) A hollow shaft having outer diameter "n" times the inner diameter.

(ii) A solid shaft.

Masses, lengths and materials of the two shafts are the same.

<u>Solution:</u>

Let D_H = Outer diameter of the hollow shaft.

d_H = Inner diameter of the hollow shaft, and

D_S = Diameter of solid shaft.

For the same mass:

$$m_H = m_S$$

$$(\text{Volume})_H \times \rho_H = (\text{Volume})_S \times \rho_S \quad \text{--- (1)}$$

It is given that, the material for solid and hollow shafts same. So,

$$f_S = f_H = f.$$

Length of solid and hollow ~~shafts~~ shafts same.

So, $l_S = l_H = l.$

WKT from (1);

$$\Rightarrow (Volume)_H \times f_H = (Volume)_S \times f_S$$

$$\Rightarrow \frac{\pi}{4}\left(D_H^2 - d_H^2\right) \times l \times f = \frac{\pi}{4} D_S^2 \times l \times f$$

$$\Rightarrow D_H^2 - d_H^2 = D_S^2 \text{ ———— }$$

From the pbm,

$$D_H = n d_H \quad \text{--- (2)}$$

Sub. (3) in (2);

$$n^2 d_H^2 - d_H^2 = D_S^2$$

$$D_S^2 = (n^2 - 1) d_H^2 \quad \text{--- (3)}$$

$$U_{hollow} = \frac{I^2}{4C} \times \left(\frac{D_H^2 + d_H^2}{D_H^2} \right) \times \text{Volume}$$

$$U_{hollow} = \frac{I^2}{4C} \times \frac{D_H^2 + d_H^2}{D_H^2}$$

$$\times \frac{\pi}{4} (D_H^2 - d_H^2) \times l$$

$$U_{hollow} = \frac{I^2}{4C} \times \frac{\pi}{4} \times l \times \frac{D_H^4 - d_H^4}{D_H^2} \quad \text{--- (4)}$$

$$U_{solid} = \frac{I^2}{4C} \times \frac{\pi}{4} D_S^2 l \quad \text{--- (5)}$$

$$\frac{U_{hollow}}{U_{solid}} = \frac{\frac{\tau^2}{4C} \times \frac{\pi}{4} \times l \left(\frac{D_H^4 - d_H^4}{D_H^2}\right)}{\frac{\tau^2}{4C} \times \frac{\pi}{4} \times D_S^2 \times L}$$

$$\frac{U_{hollow}}{U_{solid}} = \frac{D_H^4 - d_H^4}{D_H^2 \cdot D_S^2} \quad \text{——— (6)}$$

Sub. (2) in (6);

$$\frac{U_{hollow}}{U_{solid}} = \frac{n^4 d_H^4 - d_H^4}{n d_H^2 D_S^2} \quad \text{——— (7)}$$

Sub. (3) in (7);

$$\frac{U_{Hollow}}{U_{solid}} = \frac{(n^4 - 1) d_H^4}{n^2 d_H^2 \times (n^2 - 1) d_H^2}$$

$$\frac{U_{Hollow}}{U_{solid}} = \frac{(n^4 - 1)}{n^2 (n^2 - 1)}$$

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$$\frac{U_{Hollow}}{U_{solid}} = \frac{(n^2-1)(n^2+1)}{n^2(n^2-1)}$$

$$\frac{U_{Hollow}}{U_{solid}} = \frac{n^2+1}{n^2} = 1+\frac{1}{n^2}$$

5.I Therefore, hollow shaft is able to absorb more strain energy as compared to a Solid shaft.

$\frac{U_{hollow}}{U_{solid}} = 1$ to 2 for all shafts under the Conditions of the problem.

5.7 The external diameter of hollow shaft is "n" times its internal diameter. It transmits a torque T_H and develops Strain ~~eng~~ energy U_H. Another Solid Shaft has the Same external diameter as the hollow Shaft and transmits torque T_S and develops a strain energy U_S.

Find the ratios $\frac{U_H}{U_S}$ and $\frac{T_H}{T_S}$ if the two Shafts are to be Subjected to the Same max. Shear stress. Assume the ~~Shafts~~ to be of the Same material and of the Same length. Hence Show that, $\frac{T_H}{T_S} = \frac{U_H}{U_S}$.

Solution:

Let, D_H = External diameter of the hollow Shaft

d_H = Internal diameter of the hollow shaft.

$$(D_H = n\, d_H) \text{ ——— } (1)$$

D_s = diameter of Solid shaft

$$D_s = D_H \quad \text{(Given)} \text{ ——— } (2)$$

Now, For Hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau \times \left(\frac{D_H^4 - d_H^4}{D_H}\right) \text{ ——— } (3)$$

For ~~Solid S~~ Solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times D_S^3 \text{ ——— } (4)$$

$$\frac{T_H}{T_S} = \frac{\frac{\pi}{16} \times \tau \times \left(\frac{D_H^4 - d_H^4}{D_H}\right)}{\frac{\pi}{16} \times \tau \times D_S^3}$$

$$\frac{T_H}{T_S} = \frac{D_H^4 - d_H^4}{D_H \cdot D_S^3} \text{ ——— } (4)$$

Sub. (2) in (4);

$$\frac{T_H}{T_S} = \frac{D_H^4 - d_H^4}{D_H \times D_H^3}$$

$$\frac{T_H}{T_S} = \frac{D_H^4 - d_H^4}{D_H^4} \longrightarrow (5)$$

Sub. (1) in (5);

$$\frac{T_H}{T_S} = \frac{n^4 d_H^4 - d_H^4}{n^4 d_H^4}$$

$$\frac{T_H}{T_S} = \frac{(n^4 - 1) d_H^4}{n^4 d_H^4} = \frac{n^4 - 1}{n^4}$$

$$\boxed{\frac{T_H}{T_S} = \frac{n^4 - 1}{n^4}} \longrightarrow (5a)$$

WKT, $U_H = \frac{I^2}{4C} \times \left(\frac{D_H^4 + d_H^4}{D_H^2}\right) \times \text{Volume}$

$$U_H = \frac{I^2}{4C} \times \left(\frac{D_H^4 + d_H^4}{D_H^2}\right) \times \frac{\pi}{4}\left(D_H^2 - d_H^2\right) \times l$$

$$U_H = \frac{I^2}{4C} \times \frac{\pi}{4} \times \left(\frac{D_H^4 - d_H^4}{D_H^2}\right) \times l \quad \text{—— (6)}$$

$$U_s = \frac{I^2}{4C} \times \frac{\pi}{4} D_s^2 l$$

$$\boxed{D_s = D_H}$$

$$\therefore \quad U_s = \frac{I^2}{4C} \times \frac{\pi}{4} D_H^2 l \quad \text{—— (7)}$$

$$\frac{U_H}{U_s} = \frac{\frac{I^2}{4C} \times \frac{\pi}{4} \times \left(\frac{D_H^4 - d_H^4}{D_H^2}\right) \times l}{\frac{I^2}{4C} \times \frac{\pi}{4} \times D_H^2 \times l}$$

$$\frac{U_H}{U_s} = \frac{D_H^4 - d_H^4}{D_H^2 \times D_H^2} = \frac{D_H^4 - d_H^4}{D_H^4} \quad \text{—— (8)}$$

sub. (1) in (8);

$$\frac{U_H}{U_S} = \frac{(n^4-1)d_H^4}{n^4 \cdot d_H^4} = \frac{n^4-1}{n^4}$$

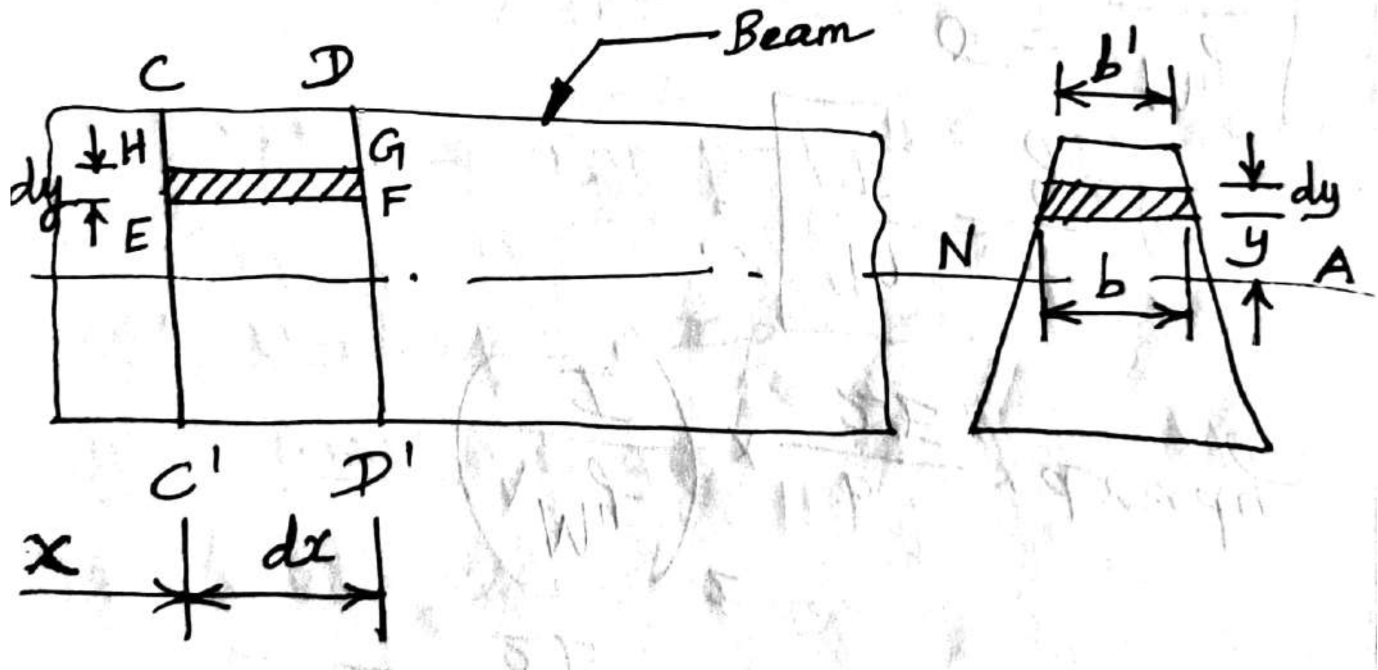
$$\boxed{\frac{U_H}{U_S} = \frac{n^4-1}{n^4}} \quad \text{(8a)}$$

From (5a) & (8a);

$$\boxed{\frac{T_H}{T_S} = \frac{U_H}{U_S}}$$

Strain energy due to Bending:

Figure shows a beam of uniform Cross-section with Certain end Conditions such that the bending moment varies along its length. Consider a small length dx of a beam where the bending moment is M. Consider further a small strip EFGH of thickness dy at a distance "y" from the neutral axis. Let "b" be the width of the strip. Volume of the strip is dx.dy.b.



∴ strain energy of the small volume

$$(dx \cdot dy \cdot b) = \frac{(\text{stress on } EFGH)^2}{2E} \times \text{Volume}$$

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Strain energy of the small Volume

$$= \frac{\sigma^2}{2E} \times b \times dx \cdot dy$$

$$= \left(\frac{My}{I}\right)^2 \times \frac{b}{2E} \times dx \times dy$$

$$\because \left[\frac{M}{I} = \frac{\sigma}{y} \text{ (or) } \sigma = \frac{My}{I}\right]$$

$$= \frac{M^2}{2EI^2} by^2 \cdot dx \cdot dy$$

dU = strain energy of the Volume within $CC'D'D$.

$$= \int \left(\frac{M^2 dx}{2EI^2}\right) (y^2 \cdot b \cdot dy)$$

$$= \frac{M^2 dx}{2EI^2} \left[\int y^2 \, b \cdot dy\right] \quad (1)$$

WKT,

$$\int b y^2 dy = \text{Sum of Second moments of areas } b.dy$$

$$\int b y^2 dy = \text{Moment of Inertia of Cross-Section} = I \quad \text{—— (2)}$$

Sub. (2) in (1);

$$dU = \frac{M^2 dx}{2EI^2} \times I$$

$$dU = \frac{M^2}{2EI} \cdot dx \quad \text{—— (3)}$$

The above expression gives the Strain energy of the beam of length dx.

$\therefore$ Strain energy of the whole of the beam of length dx,

$$U = \int \frac{M^2}{2EI} dx \quad \text{--- (3)}$$

For any given load and end Conditions, M Can be expressed in terms of x and then the total strain energy Can be evaluated with help of eqn (3);

In Case M is Constant over the length, l,

$$\boxed{U = \frac{M^2 l}{2EI}} \quad \text{--- (4)}$$

<u>strain energy and deflection due to bending:</u>

In order to Calculate the deflection under the load in the Cases of beams under the action of a Single point load, after Calculating the strain energy of beam, it is equated to the workdone by that load for its gradual ~~mov~~ movement equal to the deflection. If y is the deflection under the load W then,

$$U = \frac{1}{2} W y.$$

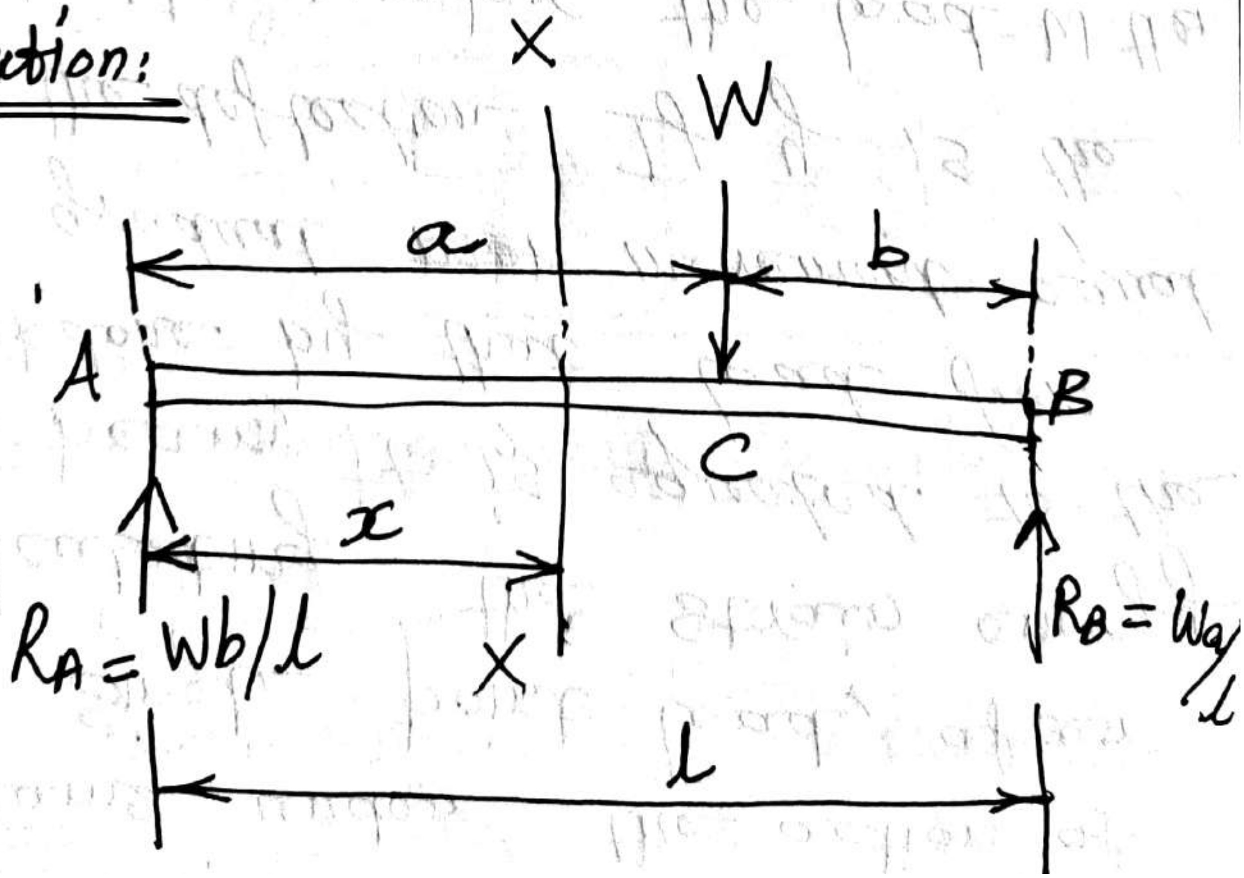
(or) $$\boxed{y = \frac{2U}{W}}$$

(5.8) A beam of length "l" simply supported at the ~~ends~~ ends is loaded with a point load W at a distance "a" from one end. ~~A~~ Assuming that the beam has

Constant Cross-Section with moment of inertia as I and Young's modulus of elasticity for the material of the beam as E, find the strain energy of the beam and hence find the deflection under the load.

Strain energy due to shearing may be neglected.

<u>Solution:</u>



To find reaction R_B, take moments about A, (#5)

$$\sum M_A = 0 \quad (\text{CCW} \Rightarrow +\text{ve})$$

$$R_B \times l - Wa = 0$$

$$\therefore \boxed{R_B = \frac{Wa}{l}}$$

~~Ra~~ Upward forces = Downward forces

$$R_A + R_B = W \qquad \boxed{\because b = l - a}$$

$$R_A + \frac{Wa}{l} = W$$

$$R_A = W - \frac{Wa}{l} \cancel{= W\left(\frac{l-a}{l}\right)}$$

$$R_A = \frac{Wl - Wa}{l} = \frac{W}{l}(l-a)$$

$$\boxed{R_A = \frac{Wb}{l}} \qquad \boxed{R_A = Wb/l}$$

For any Section XX lying between A and C; $0 < x < a$;

$$M_x = R_A \cdot x = \frac{Wb}{l} \cdot x$$

and for $a < x < l$

$$M_x = R_B (l - x) = \frac{Wa}{l}(l - x)$$

strain energy,

$$U = U_{AC} + U_{BC}$$

$$U = \int_0^a \frac{1}{2EI} M^2 \, dx + \int_a^l \frac{1}{2EI} M^2 \, dx$$

$$U = \int_0^a \frac{1}{2EI} \times \frac{W^2 b^2}{l^2} x^2 \, dx$$

$$+ \int_a^l \frac{1}{2EI} \times \frac{W^2 a^2}{l^2} (l - x)^2 \, dx$$

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$$U = \left[\frac{W^2b^2}{2EIl^2}\left(\frac{x^3}{3}\right)\right]_0^a + \left[\frac{W^2a^2}{2EIl^2} \times \left(\frac{(l-x)^3}{3}\right) \times (-1)\right]_a^l$$

$$U = \left[\frac{W^2b^2}{2EIl^2}\left[\frac{a^3}{3}\right] - 0\right] + \left[\frac{W^2a^2}{2EIl^2}\left[-\frac{(l-l)^3}{3} + \frac{(l-a)^3}{3}\right]\right]$$

$$U = \left[\left(\frac{W^2b^2}{2EIl^2} \times \frac{a^3}{3}\right) + \left[\frac{W^2a^2}{2EIl^2} \times \frac{(l-a)^3}{3}\right]\right]$$

$$U = \left[\frac{W^2b^2a^3}{6EIl^2} + \frac{W^2a^2(l-a)^3}{6EIl^2}\right]$$

$$U = \frac{W^2 b^2 a^3}{6EIl^2} + \frac{W^2 a^2}{6EIl^2}\left[(l-a)^2(l-a)\right]$$

$$\boxed{l-a=b}$$

$$U = \frac{W^2 b^2 a^3}{6EIl^2} + \frac{W^2 a^2 b^2}{6EIl^2} \times (l-a)$$

$$U = \frac{W^2 a^2 b^2}{6EIl^2}\left[a + (l-a)\right]$$

$$U = \frac{W^2 a^2 b^2 l}{6EIl^2}$$

$$\boxed{U = \frac{W^2 a^2 b^2}{6EIl}}$$

Let y_c = Deflection under load,

$\therefore$ Workdone by the load W

on the beam $= \frac{1}{2} W . y_c$

Since, the Workdone

$= $ Strain energy

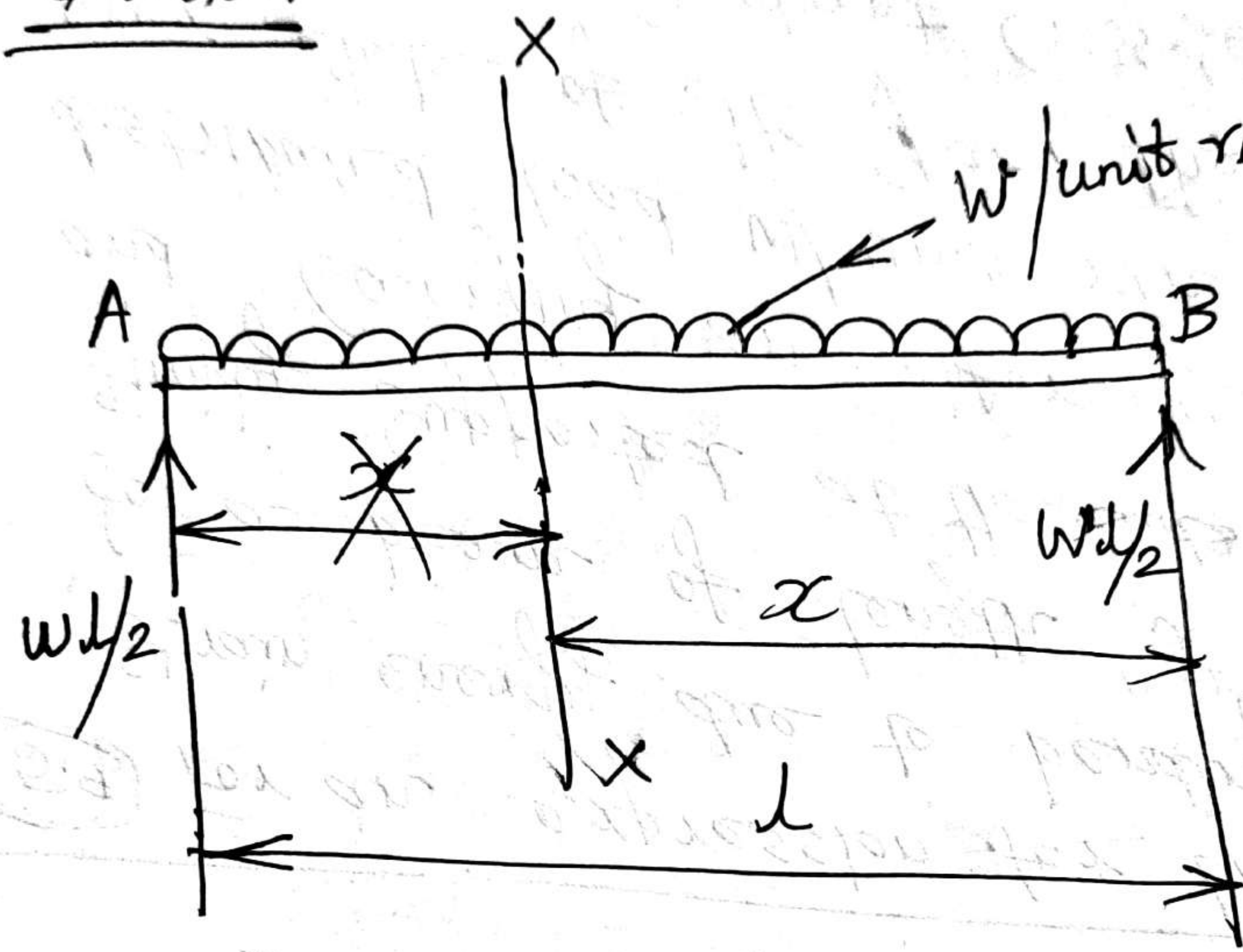
$$\therefore \frac{1}{2} W \times y_c = \frac{Wa^2b^2}{6EIl}$$

$$\therefore \boxed{y_c = \frac{Wa^2b^2}{3EIl}}$$

(5.9) For an expression for the Strain energy due to bending for a beam of length "l" Simply Supported at the ends and Carrying an uniformly distributed load w/ unit run over whole of its Span. The beam is of Constant Cross-Section throughout its length having =

flexural rigidity as EI.

Solution:



Consider any section XX at a distance "x" from the end A. The bending moment at the section is given as,

$$M_x = \frac{wl}{2} \times x - wx \cdot \frac{x}{2}$$

$$M_x = \frac{Wlx}{2} - \frac{Wx^2}{2} \quad \text{—— (1)}$$

Strain energy, $U = \int_0^l \frac{M^2}{2EI} dx$ —— (2)

$$U = \int_0^l \left(\frac{wl}{2} \cdot x - \frac{wx^2}{2} \right)^2 dx \times \frac{1}{2EI}$$

$$U = \frac{1}{2EI} \int_0^l \left(\frac{w^2 l^2 x^2}{4} + \frac{w^2 x^4}{4} - \frac{2w^2 l x^3}{4} \right) dx$$

$$U = \frac{1}{2EI} \left[\frac{w^2 l^2}{4} \times \frac{x^3}{3} + \frac{w^2 \cdot x^5}{5 \times 4} - \frac{2w^2 l x^4}{4 \times 4} \right]_0^l$$

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$$U = \frac{w^2}{8EI}\left[\frac{l^2x^3}{3} + \frac{x^5}{5} - \frac{2lx^4}{4}\right]_0^l$$

$$U = \frac{w^2}{8EI}\left[\frac{l^2l^3}{3} + \frac{l^5}{5} - \frac{2l\cdot l^4}{4}\right]$$

$$U = \frac{w^2}{8EI}\left[\frac{l^5}{3} + \frac{l^5}{5} \cancel{3} - \frac{2l^5}{4}\right]$$

$$U = \frac{w^2}{8EI}\left[\frac{l^5}{3} + \frac{l^5}{5} - \frac{l^5}{2}\right]$$

$$U = \frac{w^2}{8EI}\left[\frac{10l^5 + 6l^5 - 15l^5}{\cancel{30}\ 30}\right]$$

$$U = \frac{w^2}{8EI}\left[\frac{l^5}{30}\right]$$

$$\boxed{U = \frac{w^2 l^5}{240 EI}}$$

Castigliano's theorem:

Castigliano's theorem can be used in the following cases:

1. To determine the displacements of complicated structures.
2. To find the deflection of beams due to shearing (or) bending if the total strain energy due to shearing forces (or) bending moments (as the ~~s~~ case may be) is known.
3. To find the ~~defect~~ deflections of curved beams, springs etc.

Castigliono's theorem is stated as follows:

"If U is the total strain energy of any structure due to the application of external loads $W_1, W_2, W_3, \ldots W_n$ at points $A_1, A_2, A_3 \ldots A_n$ respectively in the direction $AX_1, AX_2, AX_3 \ldots AX_n$ and due to Couples $M_1, M_2, \ldots M_n$, at points $B_1, B_2, B_3, \ldots B_m$ respectively then the deflection at the points $A_1, A_2, A_3, \ldots A_n$ in the ~~directions~~ directions $AX_1, AX_2, AX_3 \ldots AX_n$ are $\frac{\partial U}{\partial W_1}, \frac{\partial U}{\partial W_2}, \frac{\partial U}{\partial W_3}, \ldots \frac{\partial U}{\partial W_n}$, and the angular positions of the Couples are $\frac{\partial U}{\partial M_1}, \frac{\partial U}{\partial M_2}, \frac{\partial U}{\partial M_3}$

$\ldots \frac{\partial U}{\partial M_n}$ at the respective points of application".

Points to remember while applying Castigliano's theorem:

① (a) Treat all the loads and Couples/moments as Variables and Carry out partial differentiation.

(b) Substitute the numerical Values of different loads and Couples in the above equation.

② To findout the deflection (or) rotation at a point of the structure where there is no load (or) Couple acting, then it may be assumed that a dummy load W (or) dummy moment/Couple is acting at that point and give a value zero at the end:

i.e., $x = \left(\frac{\partial U}{\partial W}\right)_{W=0}$

and $\phi = \left(\frac{\partial U}{\partial M}\right)_{M=0}$

(5.10) Using Castigliano's theorem, obtain the deflection under a single concentrated load applied to a simply supported beam as shown in figure. $EI = 2.2\,MNm^2$.

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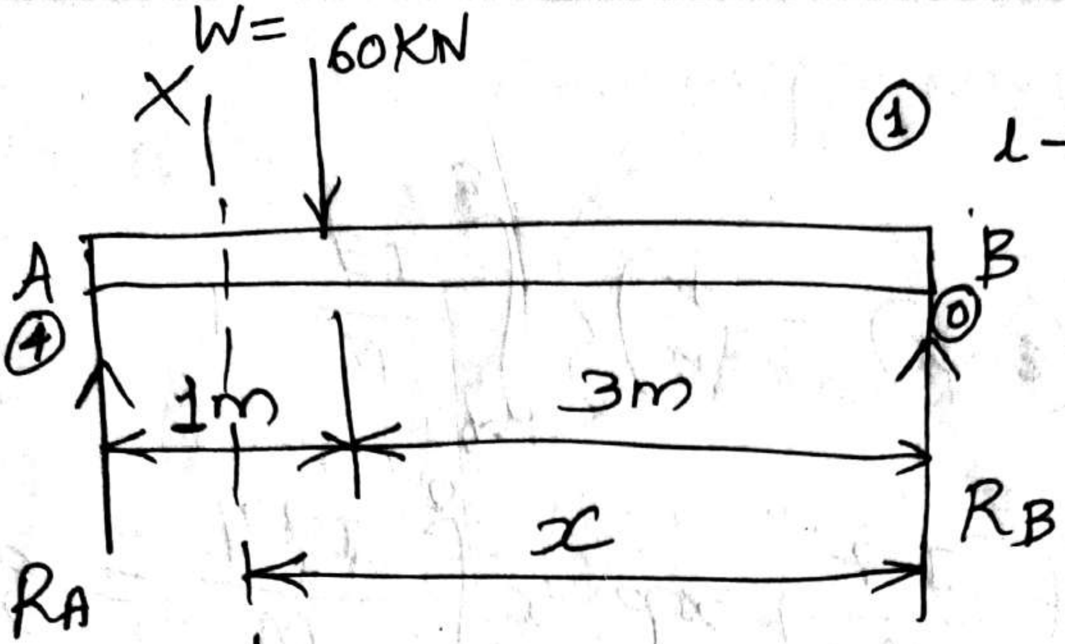
(1) $l \to 3$ to 4

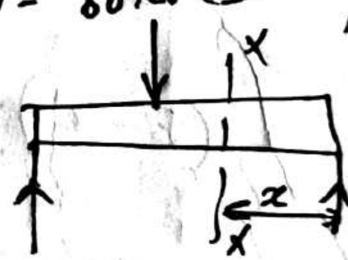


$$\Sigma M_A = 0 \quad (CCW \Rightarrow +ve)$$

$$R_B \times 4 - W \times 1 = 0$$

$$\boxed{R_B = \frac{W}{4}}$$

(2) : $l \to 0$ to 3 :



$M_x = R_B x$

$M_x = \frac{Wx}{4}$

$\frac{\partial m}{\partial W} = \frac{x}{4}$.

Consider a Section XX at a distance x from B;

$$M_x = R_B \times x - W(x-3)$$

$$\boxed{M_x = \frac{Wx}{4} - W(x-3)} \quad (1)$$

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$$\boxed{\frac{\partial M}{\partial W} = \frac{x}{4} - (x-3)} \qquad (2)$$

Now, $\delta = \int_0^l \frac{M}{EI} \cdot \frac{\partial m}{\partial W}\, dx,$

$$\delta = \frac{1}{EI}\int_0^3 \left(\frac{Wx}{4} \times \frac{x}{4}\, dx\right)$$

$$+ \frac{1}{EI}\int_3^4 \left\{\frac{Wx}{4} - W(x-3)\right\} \times \left\{\frac{x}{4} - (x-3)\right\} dx$$

$$\delta = \frac{W}{16EI}\int_0^3 x^2\, dx \cancel{+ \frac{W}{EI}\int_3^4}$$

$$+ \frac{W}{EI}\int_3^4 \left\{\left(\frac{x}{4} - (x-3)\right)\right\}^2 dx$$

$$\delta = \frac{W}{16EI}\int_0^3 x^2\,dx + \frac{W}{EI}\left[\int_4^3 \left(\frac{x-4x+12}{4}\right)^2 dx\right]$$

$$\delta = \frac{W}{16EI}\int_0^3 x^2\,dx + \frac{9W}{16EI}\int_3^4 (x^2-8x+16)\,dx$$

$$\delta = \frac{W}{16EI}\left[\frac{x^3}{3}\right]_0^3 + \frac{9W}{16EI}\left[\frac{x^3}{3} - \frac{8x^2}{2} + 16x\right]_4^3$$

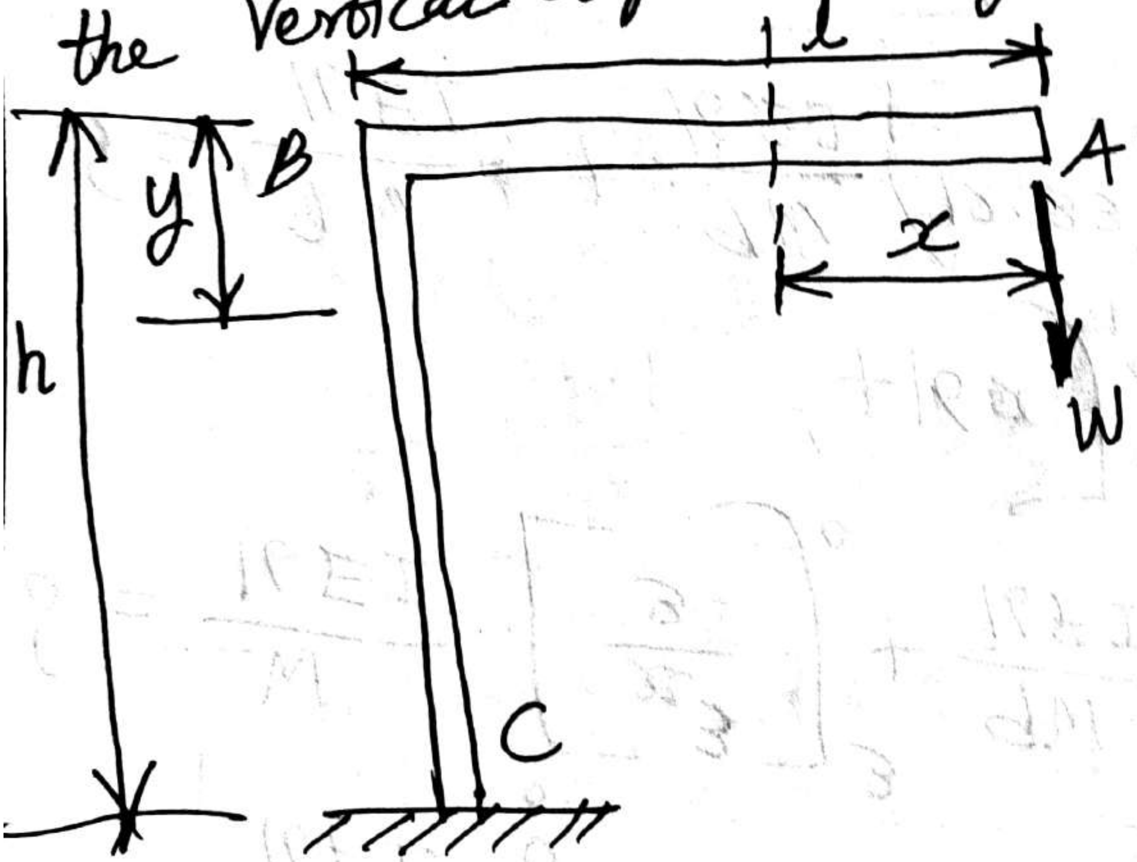
$$\delta = \frac{9W}{16EI} + \frac{9W}{16EI}(12.33 - 28 + 16)$$

$$\delta = \frac{9W}{16EI}\left[1 + 0.33\right] = \frac{0.75W}{EI}$$

$$\delta = \frac{0.75 \times 60 \times 10^3}{2.2 \times 10^6} = 0.020450\ m$$

$$\boxed{\delta = 20.45\ mm}$$

(5.11) In the figure, which shown a structure. Assuming the member to be of uniform Cross-Section throughout find the strain energy stored by the structure and hence determine the Vertical deflection of end A.



Solution: —

Section AB:

$$M_x = W \cdot x$$

$$U_{AB} = \int_0^l \frac{M_x^2\,dx}{2EI}$$

$$U_{AB} = \int_0^l \frac{W^2 x^2\,dx}{2EI} = \frac{W^2 l^3}{6EI}$$

Section BC:

$$M_y = W \cdot l$$

$$U_{BC} = \int_0^h \frac{M_y^2\,dy}{2EI} = \int_0^h \frac{W^2 l^2\,dy}{2EI}$$

$$U_{BC} = \frac{W^2 l^2 h}{2EI}$$

~~Secti~~ Total strain energy:

$$U = U_{AB} + U_{BC}$$

$$U = \frac{W^2 l^3}{6EI} + \frac{W^2 l^2 h}{2EI}$$

$$\boxed{U = \frac{W^2 l^2}{6EI}(l+3h)}$$

Let δ be the deflection of end A. Then, ~~W~~

Workdone by W = Total strain energy stored.

$$\frac{1}{2} W \delta = \frac{W^2 l^2}{6EI}(l+3h)$$

$$\boxed{\delta = \frac{W l^2}{3EI}(l+3h)}$$